No.

Date

Summary: All Theorems, Lemma, Corollary of MC

All Theorems in Durette.

T1. Strong Markov Property: Suppose T is stopping time. Given $T=n$ and $X_T = y$, and other information about $X_0, \cdots, X_T$ is irrelevant for predicting the future, and X_{T+k}, $k \geq 0$, behaves like MC with initial state y.

保证有限步回来

T2. If $\rho_{xy} = P_x(T_y < \infty) > 0$, but $\rho_{yx} < 1$, then x is transient

T3. If C is a finite closed and irreducible set, then all states in C are recurrent.

T4. If S is finite, S can be written as $T \cup R_1 \cup \cdots$, where T is set of transients and R_i are closed irreducible sets of recurrent states.

T5. y is recurrent iff $\sum_{n=1}^{\infty} p_{yy}^{(n)} = E_y N(y) = \infty$

where $N(y)$ be the # of visits to y at times $n \geq 1$

补: $\lim_{n\to\infty} p_{yy}^{(n)} = 1/\sum_{n=0}^{\infty} n f_{yy}^{(n)}$，即$n\to\infty$下在y概率为期望首次返回y所用时间倒数

T6. If p is doubly stochastic transition prob matrix for a MC, then $\pi(x) = 1/N$ is the unique stationary distribution.

T7. Convergence Theorem: If I, A, S, then $n \to \infty$, $p_{xy}^{(n)} \to \pi(y)$

- I: p is irreducible — i.e., limiting dist exists and equals to stat dist
- A: aperiodic — *: I & A & finite $\Leftrightarrow$ Regular: $\exists k_0, \forall k > k_0, p^k$ all 元素 >0
- R: all states are recurrent
- S: there's a stationary dis

T8. If I & R, $\frac{N_n(y)}{n} \to \frac{1}{E_y T_y}$

T9. If I & S, $\pi(y) = 1/E_y T_y$

T10. Suppose I, S, and $\sum_x |f(x)| \pi(x) < \infty$, then $\frac{1}{n}\sum_{m=1}^{n} f(X_m) \to \sum_x f(x)\pi(x)$

T11. Suppose I, S: $\frac{1}{n}\sum_{m=1}^{n} p_{xy}^{m} \to \pi(y)$

T12. Suppose p is irreducible & recurrent, $n \to \infty$: $\frac{N_n(y)}{n} \to \frac{1}{E_y T_y}$ (与T8重复)

T13. $\mu_x(y)$: 在x回到x之前平均访问y的次数:

$$\mu_x(y) = \sum_{n=0}^{\infty} P_x(X_n = y, T_x > n)$$，是 stationary measure

i.e., $\mu = (\mu_x(s_1), \mu_x(s_2), \cdots)$ 与 π 呈倍数关系

T14. Suppose P is I, then: some states are positive recurrent $\Leftrightarrow$ all are $\Leftrightarrow$ has a stat dist π

补: If $0 < p_i < 1$, $i = 0, 1, \cdots$, then $u_m = \prod_{i=0}^{m}(1-p_i) \to 0$ as $m \to 0$ iff $\sum_{i=0}^{\infty} p_i = \infty$

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All Lemmas in Durette:

L1. Suppose $P_x(T_y \leq k) \geq \alpha > 0$ for $\forall x$, then: $P_x(T_y > nk) \leq (1-\alpha)^n$

L2. If $x \to y$, $y \to z$, then $x \to z$

L3. If x is recurrent and $\rho_{xy} > 0$, then $\rho_{yx} = 1$

L4. If x is recurrent and $x \to y$, then y is recurrent

L5. In a finite closed set there has to be at least one recurrent state

L6. $E_x N(y) = \rho_{xy} / (1-\rho_{yy})$ L7. $E_x N(y) = \sum_{n=1}^{\infty} p_{xy}^{(n)}$

L8. If $\rho_{xy}^{*} > 0$ and $\rho_{yx} > 0$ then x, y have same period

$\rho_{xy} \Leftrightarrow x \to y \Leftrightarrow \exists k,\ P_{xy}^{(k)} > 0$

L9. If $P_{xx} > 0$, then x has period 1

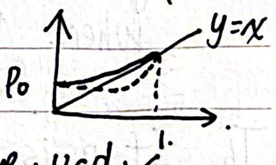
L10. If there's a stat dist, then all states y that have $\pi(y) > 0$ are recurrent

L11. The extinction probability ρ is the smallest solution of $\phi(x) = x$ with $x \in [0,1]$

L12. Suppose $\rho = \phi(\rho)$ $\rho \in [0,1]$, then if $P_1 < 1$, then $\lim_{t \to \infty} X_t = \begin{cases} \infty, & P = 1-\rho \\ 0, & P = \rho \end{cases}$

L13. Suppose $p_1 < 1$, $E(S) = \mu \leq 1$. $X_0 = 1$, the $Pr\{\lim_{n \to \infty} X_n = 0\} = 1$

$\left(0 \leq \frac{d\phi(s)}{d(s)}\Big|_{s=x} \overset{*}{\leq} \frac{d\phi(s)}{ds}\Big|_{s=1} = \mu \leq 1,\ *: = p_1''\text{导} = 2p_2 \geq 0\right)$



L14. Finite irreducible $\Rightarrow$ recurrent! 但 Infinite 不一定!

Finite irreducible $\Rightarrow$ Must Unique Stat Dist Exist; Infinite $\Rightarrow \begin{cases} \text{positive: usd } \checkmark \\ \text{null/tran: no} \end{cases}$

Aperiodic or not $\Rightarrow$ 'Limiting dist = unique stat dist' or not