概率论

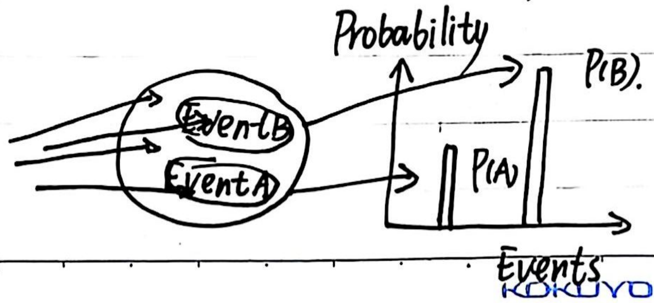
L0 前置知识

- Set A, B, 有以下 notational convention:
  空集 $\varnothing$; $A \subseteq B$; $A \cup B$ (Union)
  $A \cap B$ (Intersection); A^c: complement of A
- De Morgan's laws: $(A \cup B)^c = A^c \cap B^c$
  $(A \cap B)^c = A^c \cup B^c$
- Venn Diagram 维恩图
- Sample Space & Event (样本空间&事件)
  ① sample space S of an experiment: 所有可能结果
  ② 事件 event: subset of S
  ③ 称: 事件A发生 (occurred) if 实际结果包含在A内
- Translations Between English & Sets

sample space	S	A or B	$A \cup B$
s is a possible outcome	$s \in S$	A and B	$A \cap B$
A is a event	$A \subseteq S$	not A	A^c
A occurred	$s_{actual} \in A$	A or B, but not both	$(A \cap B^c) \cup (A^c \cap B)$
something must happen	$s_{actual} \in S$	at least one of $A_1, \cdots, A_n$	$A_1 \cup \cdots \cup A_n$
		all of $A_1, \cdots, A_n$	$A_1 \cap \cdots \cap A_n$

Relationships between events:
A implies B $\quad A \subseteq B$
A and B are mutually exclusive $\quad A \cap B = \varnothing$
$A_1, \cdots, A_n$ are a partition of S:
$A_1 \cup A_2 \cdots \cup A_n = S$, $A_i \cap A_j = \varnothing$ for $i \neq j$

- Probabilistic Model: experiment



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Lec1 Naive Definition of Probability & Counting

Assumption 1: finite sample space

2: all outcomes occur equally likely

则定义: A是样本空间中的S, $A \subseteq S$, 则naive probability of A is:

$$P_{naive}(A) = \frac{|A|}{|S|} = \frac{\text{number of outcome favorable to } A}{\text{total number of outcomes in } S}$$

Basic Counting:

Sampling: 从集合中抽一个元素

With & Without Replacement: 抽完后放回&不放回

(不译作"替换",应为"再放置") i.e. 允不允许重复抽取

Ordered & Unordered: 顺序有无重要性

离散中介绍了四种 Counting:

	With	Without
ordered	①	②
unordered	④	③

k元素中取 n个 ①: n^k outcomes.

②: $n(n-1)\cdots(n-k+1)$ outcomes

②的应用: Generalized Birthday Problem

一共 n天, 有k人, 则有两人即以上有重复生日的人的 P?

$$P = 1 - \frac{n(n-1)(\cdots)(n-k+1)}{n^k}$$, typically: 若$P=\frac{1}{2}$, $k \approx 1.18\sqrt{n}$

k.n很大时: $\frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n-k+1}{n} = (1-\frac{1}{n})(1-\frac{2}{n})\cdots(1-\frac{k-1}{n})$

$$\approx e^{-\frac{1}{n}} e^{-\frac{2}{n}} \cdots = e^{-\frac{1}{n}\frac{k(k-1)}{2}} \approx e^{-\frac{k^2}{2n}}$$

同样, 该数学建模可用于哈希冲突 (Hash Collision)

③. $\frac{n!}{k!(n-k)!} = C_n^k = \binom{n}{k}$, 它也称为binomial coefficient

有: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

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$$(x_1+x_2+\cdots+x_r)^n = \sum_{n_1,\cdots,n_r \geq 0} \frac{n!}{n_1!\cdots n_r!} x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$$

其中：$\sum_{i=1}^{r} n_i = n$，而系数由来：

$$\binom{n}{n_1}\binom{n}{n_2}\cdots\binom{n}{n_r} = \frac{n!}{n_1!\cdots n_r!}$$

Theorem: I. $n\binom{n-1}{k-1} = k\binom{n}{k}$ ⇐ 可以用"组合方法证明"（离散中有讲）

II. Vandermonde: $\binom{m+n}{k} = \sum_{j=0}^{k}\binom{m}{j}\binom{n}{k-j}$.

④ 也称：Bose-Einstein Counting

对 $x_1,\cdots,x_n$ 个正整数，满足：$x_1+x_2+\cdots+x_n=r$

则 $(x_1,\cdots,x_n)$ 这样的向量一共有：$\binom{r-1}{n-1}$ 个

理解：r 个球，$r-1$ 个 slot，插 $n-1$ 个板，两板之间为 x_i

⇔ $x_1,\cdots,x_n$ 非负，$\sum_{i=1}^{n} x_i = r$，则有 $(x_1,\cdots,x_n)$ 向量 $\binom{n+r-1}{n-1}$ 个

理解：原先基础上补上 n 个球，保证两板间至少一个球后，所有 x_i 均扣除一个球，剩余数量就是 x_i

Eg. $x_1+x_2+x_3+x_4=88$，$x_1\geq 3$，$x_2\geq 5$，$x_3\geq 8$，$x_4\geq 10$

$\Rightarrow (x_1-3)+(x_2-5)+(x_3-8)+(x_4-10)=62$

$\therefore$ 一共 $\binom{62+4-1}{3} = \binom{65}{3}$ 种 (x_1,x_2,x_3,x_4) 取法