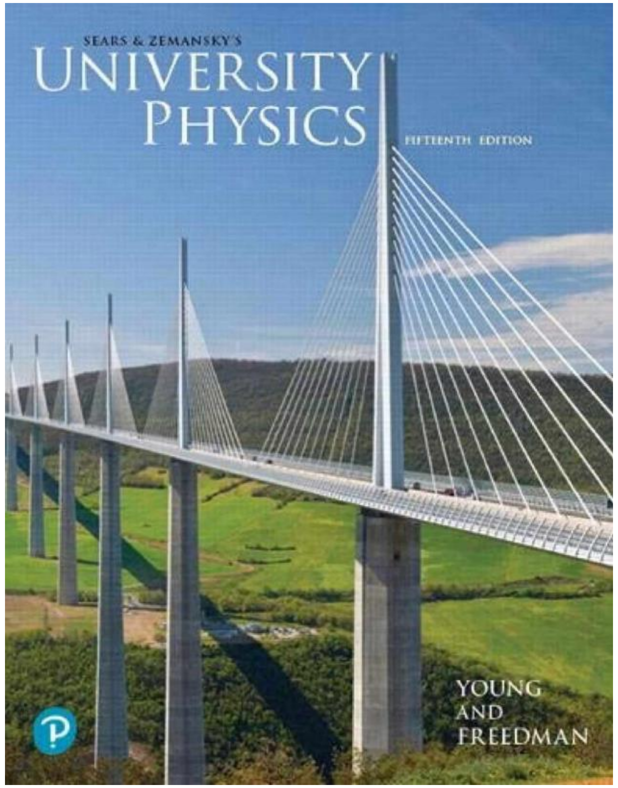
普通物理I PHYS1181 第8讲

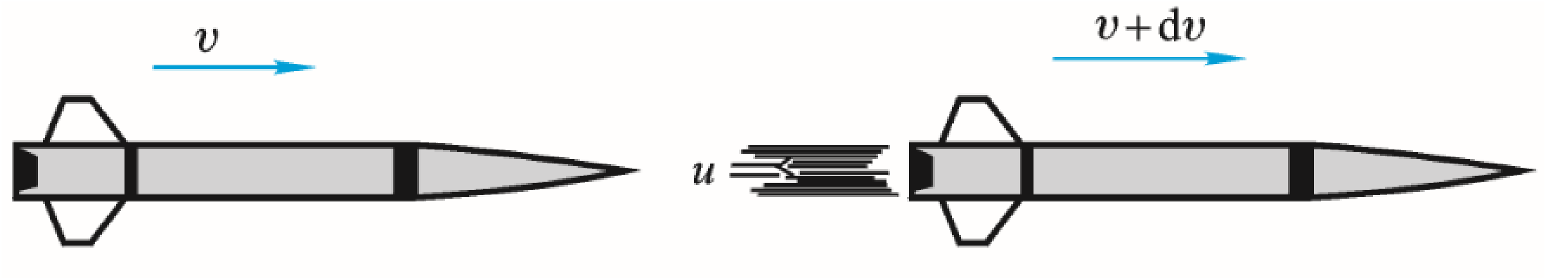
碰撞
Collision



变质量问题：火箭飞行

(1) 火箭的速度

设 t 时刻，火箭质量为 M，速度为 v (向前)，在 $\mathrm{d}t$ 内，喷出气体 $\mathrm{d}m$ (注意：$\mathrm{d}m=-\mathrm{d}M$)，喷气相对火箭的速度(称喷气速度)为 u (向后)，使火箭的速度增加了 $\mathrm{d}v$。



若不计重力和其他外力，由动量守恒定律可得

$$Mv = (M + dM)(v + \mathrm{d}\,v) + \mathrm{d}\,m\,(v + dv - u)$$
$$= (M + dM)(v + \mathrm{d}\,v) - dM(v + dv - u)$$

略去二阶无穷小量 ⇨ $\mathrm{d}v = -u\dfrac{\mathrm{d}M}{M}$

$$\mathrm{d}v = -u\frac{\mathrm{d}M}{M}$$

设*u*是一常量， $\int_{v_1}^{v_2} \mathrm{d}\, v = \int_{M_1}^{M_2} -u\frac{\mathrm{d}\, M}{M}$

$$v_2 - v_1 = u \ln\frac{M_1}{M_2}$$

设火箭开始飞行的速度为零，质量为 M_0，燃料烧尽时，火箭剩下的质量为 M，此时火箭能达到的速度是

$$v = \int_{M_0}^{M} -u\frac{\mathrm{d}\, M}{M} = u \ln\frac{M_0}{M}$$

火箭的质量比 (*N*)

多级火箭：

$$v_1 = u_1 \ln N_1$$

$$v_2 - v_1 = u_2 \ln N_2,$$

$$v_3 - v_2 = u_3 \ln N_3,$$

…………

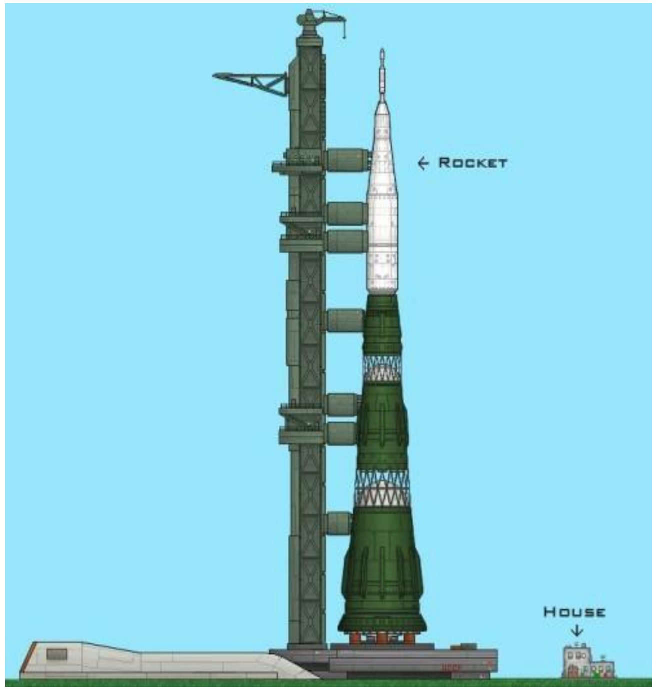
最终速度：$v_n = \sum_{i=1}^{n} u_i \ln N_i$

u_i — 第 i 级火箭喷气速率

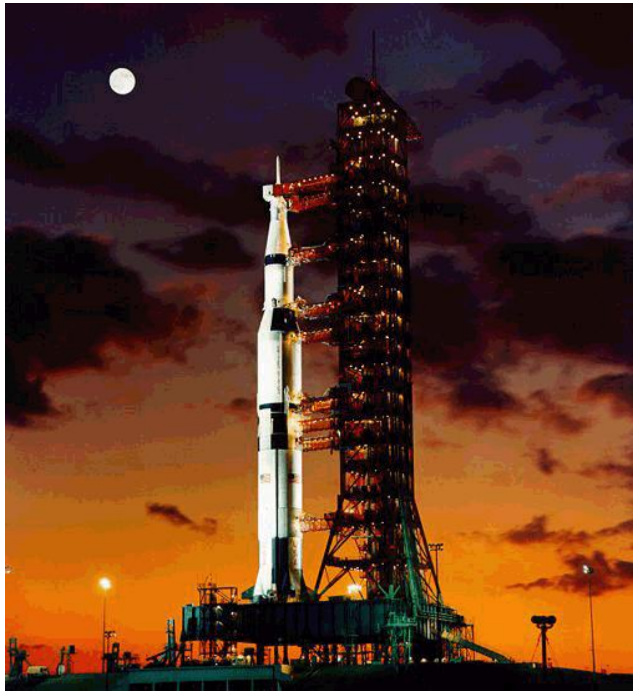
N_i — 第 i 级火箭质量比

火箭发射的“沉重”代价

苏联N1运载火箭
火箭类型：五级重型运载火箭
直径17米，高度105 米
火箭重2735吨，低地轨道载荷：75吨



美国土星5号运载火箭
火箭类型：三级液体燃料重型运载火箭
高度110.6米,直径10.1米
质量3039吨，低地轨道载荷：119吨



（2）火箭的推力

取 t 时刻喷出的燃气 $-\mathrm{d}M$ 为研究对象，其速度 v，
$t+\mathrm{d}t$ 时刻速度为 $v+\mathrm{d}v-u$

由动量定理

$$f_{\mathrm{gas}}\mathrm{d}t=(-\mathrm{d}M)(v+\mathrm{d}v-u)-(-\mathrm{d}M)v$$

略去二阶无穷小量 $\Rightarrow$ $f_{\mathrm{gas}}=u\dfrac{\mathrm{d}M}{\mathrm{d}t}$

$f_{\mathrm{gas}}<0$ 燃气受力向后

火箭的推力 $f_{\mathrm{gas}}=-u\dfrac{\mathrm{d}M}{\mathrm{d}t}$ 向前

例8-1 一空间探测器质量为6090 kg，正相对于太阳以105m/s的速率向木星运动。当它的火箭发动机相对于它以253 m/s的速率向后喷出80.0 kg的废气后，它对太阳的速率变为多少？

例8-2 在太空静止的一级火箭，点火后，其质量的减少与初质量自比为多大时，它喷出的废气将是静止的。

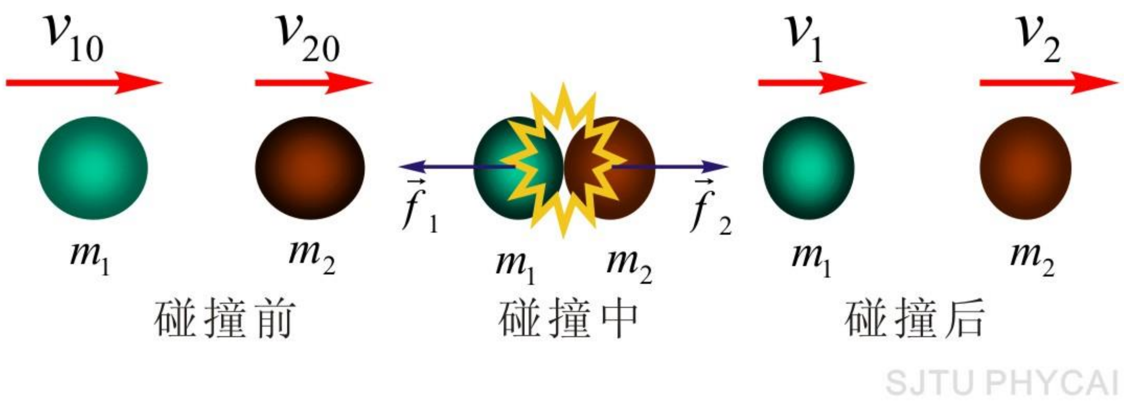
碰撞

如果两个或几个物体在相遇中，物体之间的相互作用仅持续一个极为短暂的时间，这些现象就是碰撞（collision）。如：撞击、打桩、锻铁等，以及微观粒子间的非接触相互作用过程即散射（scattering）等。

讨论两球的对心碰撞或称正碰撞（direct impact）：即碰撞前后两球的速度在两球的中心连线上。

1. 碰撞过程系统动量守恒：

$$m_1 v_{10} + m_2 v_{20} = m_1 v_1 + m_2 v_2$$



2. 牛顿的碰撞定律：碰撞后两球的分离速度(v_2-v_1)，与碰撞前两球的接近速度(v_{10}-v_{20})成正比，比值由两球的材料性质决定。即恢复系数（coefficient of restitution）：

$$e = \frac{v_2 - v_1}{v_{10} - v_{20}}$$

a. 完全弹性碰撞（perfect elastic collision）：

$e = 1 \qquad v_2 - v_1 = v_{10} - v_{20}$

b. 非弹性碰撞（inelastic collision）：

$0 < e < 1$

c. 完全非弹性碰撞（perfect inelastic collision）：

$e = 0 \qquad v_2 = v_1$

a. 完全弹性碰撞

$$e = 1 \quad \Rightarrow \quad v_1 = \frac{(m_1 - m_2)v_{10} + 2m_2 v_{20}}{m_1 + m_2}$$

$$v_2 = \frac{(m_2 - m_1)v_{20} + 2m_1 v_{10}}{m_1 + m_2}$$

机械能损失：

$$\Delta E = -\frac{1}{2}(1 - e^2)\frac{m_1 m_2}{m_1 + m_2}(v_{10} - v_{20})^2 = 0$$

完全弹性碰撞过程，系统的机械能（动能）也守恒。

b. 完全非弹性碰撞

$$e = 0 \quad \Rightarrow \quad v_1 = v_2 = \frac{m_1 v_{10} + m_2 v_{20}}{m_1 + m_2}$$

c. 非弹性碰撞

$$\begin{cases} m_1 v_{10} + m_2 v_{20} = m_1 v_1 + m_2 v_2 \\ e = \dfrac{v_2 - v_1}{v_{10} - v_{20}} \end{cases}$$

碰后两球的速度为 $v_1 = v_{10} - \dfrac{(1+e)m_2(v_{10} - v_{20})}{m_1 + m_2}$

$$v_2 = v_{20} + \frac{(1+e)m_1(v_{10} - v_{20})}{m_1 + m_2}$$

机械能损失：

$$\Delta E = -\frac{1}{2}(1-e^2)\frac{m_1 m_2}{m_1+m_2}(v_{10}-v_{20})^2$$

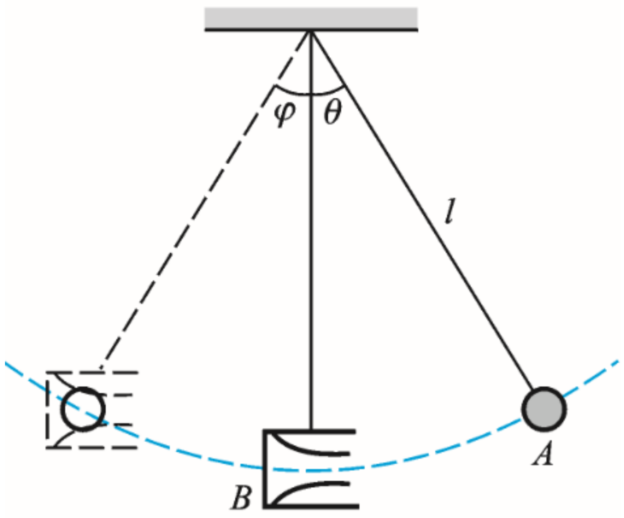
如打桩、打铁时 $v_{20}=0$

$$|\Delta E| = (1-e^2)\frac{1}{1+\frac{m_1}{m_2}}\frac{1}{2}m_1 v_{10}{}^2$$

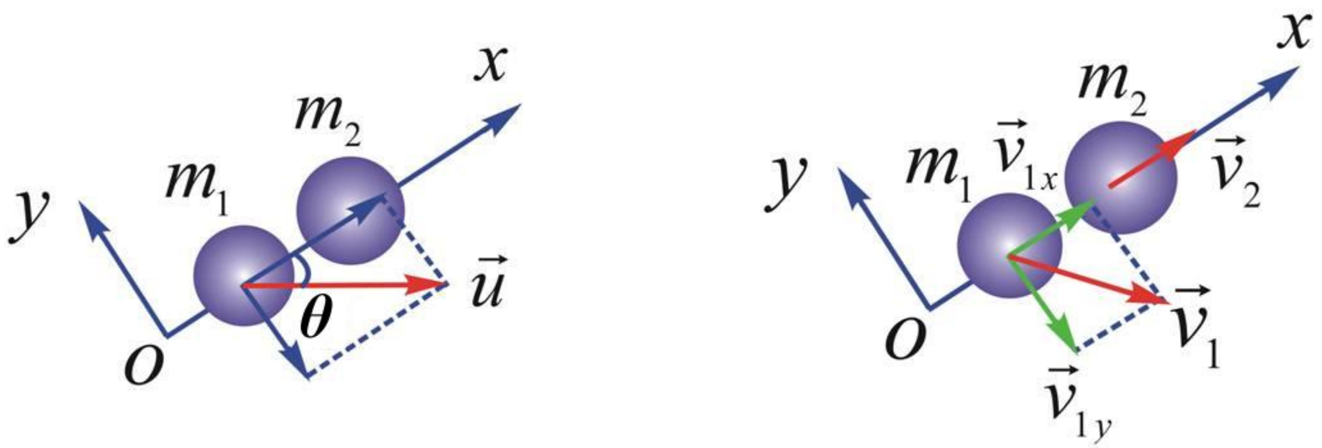
m_1/m_2越小，机械能损失越大；⇨ 打桩

m_1/m_2越大，机械能损失越小。⇨ 打铁

例8-3 如图，A为一小球，B为蹄状物，质量分别为m_1和m_2. 开始时，将A球从张角θ 处落下，然后与静止的B物相碰撞，嵌入B中一起运动，求两物到达最高处的张角 φ 。

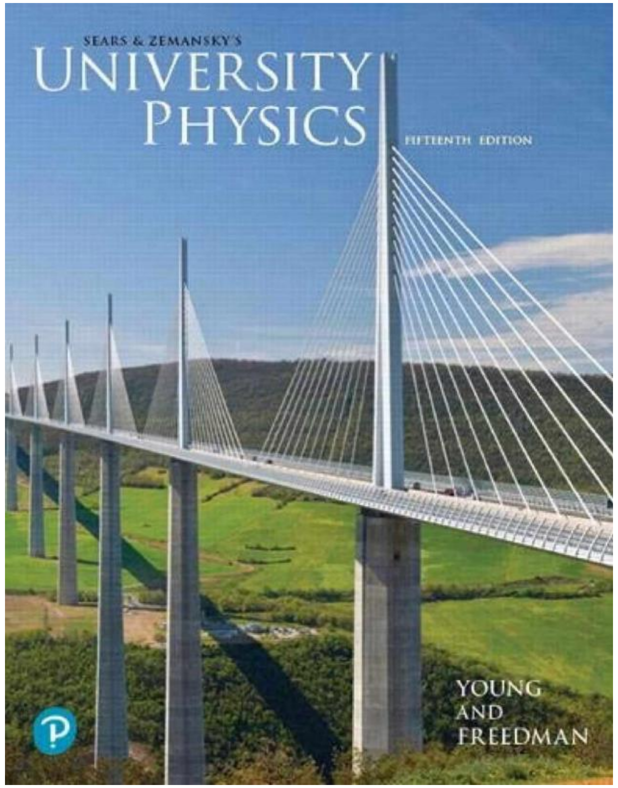


例8-4光滑桌面上，质量为m_1的小球以速度u碰在质量为m_2的静止小球上，u与两球的连心线成θ角(称为斜碰 oblique impact)。设两球表面光滑，它们相互撞击力的方向沿着两球的连心线，已知恢复系数为e，求碰撞后两球的速度。



普通物理I PHYS1181 第8讲

弹性
Elasticity



应变、应力和胡克定律

a) 拉应变和压应变

b) 体应变

c) 切应变

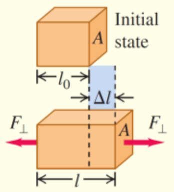
d) 胡克定律的局限性

Stress, strain, and Hooke's law: Hooke's law states that in elastic deformations, stress (force per unit area) is proportional to strain (fractional deformation). The proportionality constant is called the elastic modulus.

$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus} \quad (11.7)$$

Tensile and compressive stress: Tensile stress is tensile force per unit area, $F_{\perp}/A$. Tensile strain is fractional change in length, $\Delta l/l_0$. The elastic modulus for tension is called Young's modulus Y. Compressive stress and strain are defined in the same way. (See Example 11.5.)

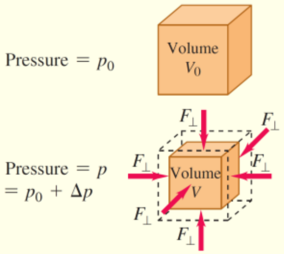
$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_{\perp}/A}{\Delta l/l_0} = \frac{F_{\perp}}{A}\frac{l_0}{\Delta l} \quad (11.10)$$



Bulk stress: Pressure in a fluid is force per unit area. Bulk stress is pressure change, Δp, and bulk strain is fractional volume change, $\Delta V/V_0$. The elastic modulus for compression is called the bulk modulus, B. Compressibility, k, is the reciprocal of bulk modulus: $k = 1/B$. (See Example 11.6.)

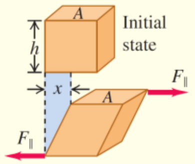
$$p = \frac{F_{\perp}}{A} \quad (11.11)$$

$$B = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V/V_0} \quad (11.13)$$



Shear stress: Shear stress is force per unit area, $F_{\parallel}/A$, for a force applied tangent to a surface. Shear strain is the displacement x of one side divided by the transverse dimension h. The elastic modulus for shear is called the shear modulus, S. (See Example 11.7.)

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A}\frac{h}{x} \quad (11.17)$$



The limits of Hooke's law: The proportional limit is the maximum stress for which stress and strain are proportional. Beyond the proportional limit, Hooke's law is not valid. The elastic limit is the stress beyond which irreversible deformation occurs. The breaking stress, or ultimate strength, is the stress at which the material breaks.

应变、应力和胡克定律

1. 什么是应变？什么是应力？

物体由于外因（受到外力、湿度、温度场变化等）而变形时，在物体内各部分之间产生相互作用的内力以抵抗形变，单位面积上的内力称为应力。

应变：物体受力后的形变； strain

$$\varepsilon_x = \frac{\Delta x}{x}$$

应力：物体单位面积上受力的大小。 stress

$$\sigma_x = \frac{F_x}{A}$$

2. 胡克定律

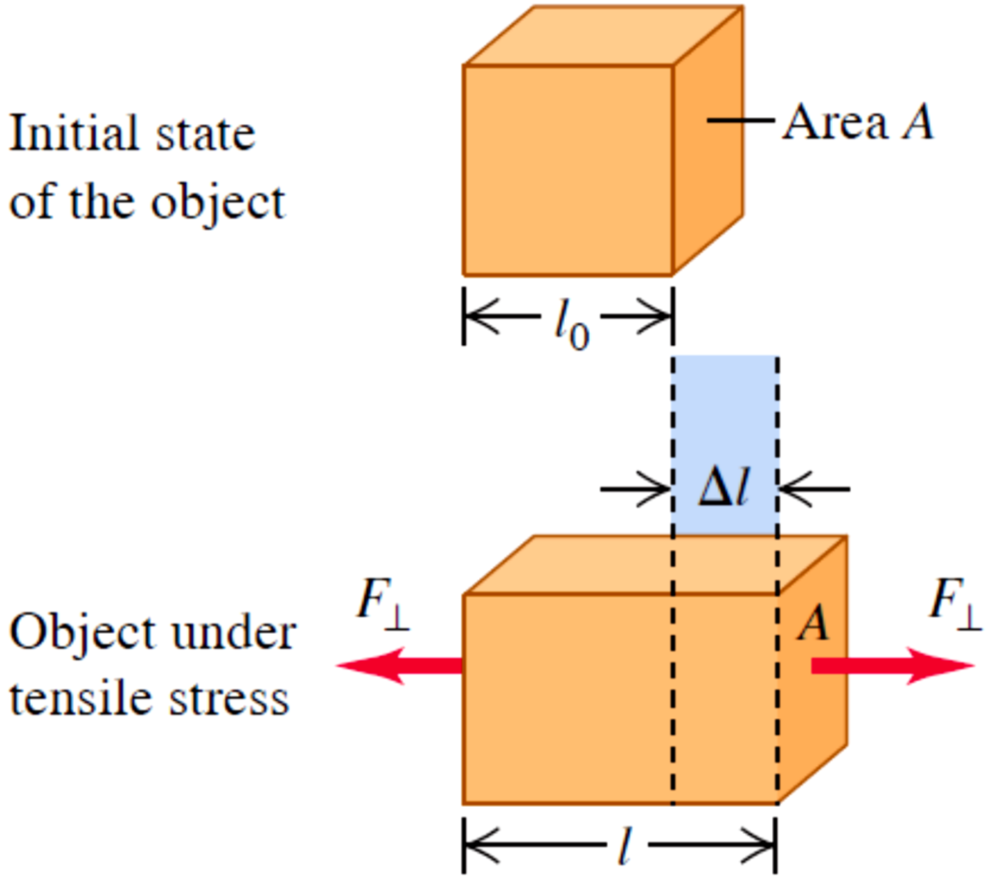
Measure of forces applied to deform an object

Hooke's law: $\dfrac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus}$

Property of material of which object is made

Measure of how much deformation results from stress

a） 拉应力/应变 Tensile stress/strain



$$\text{Tensile stress} = \frac{F_\perp}{A}$$

$$\text{Tensile strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

应力（stress）单位： Pa in SI; psi in British
应变（strain）单位： none
1 psi = 6895 Pa

弹性形变的杨氏模量Y

Young's modulus for tension

Force applied perpendicular to cross section

Original length (see Fig. 11.14)

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp/A}{\Delta l/l_0} = \frac{F_\perp}{A}\frac{l_0}{\Delta l}$$

Cross-sectional area of object

Elongation (see Fig. 11.14)

$$F_\perp = \frac{YA}{l_0}\Delta l \quad \Rightarrow \quad k = \frac{YA}{l_0}$$

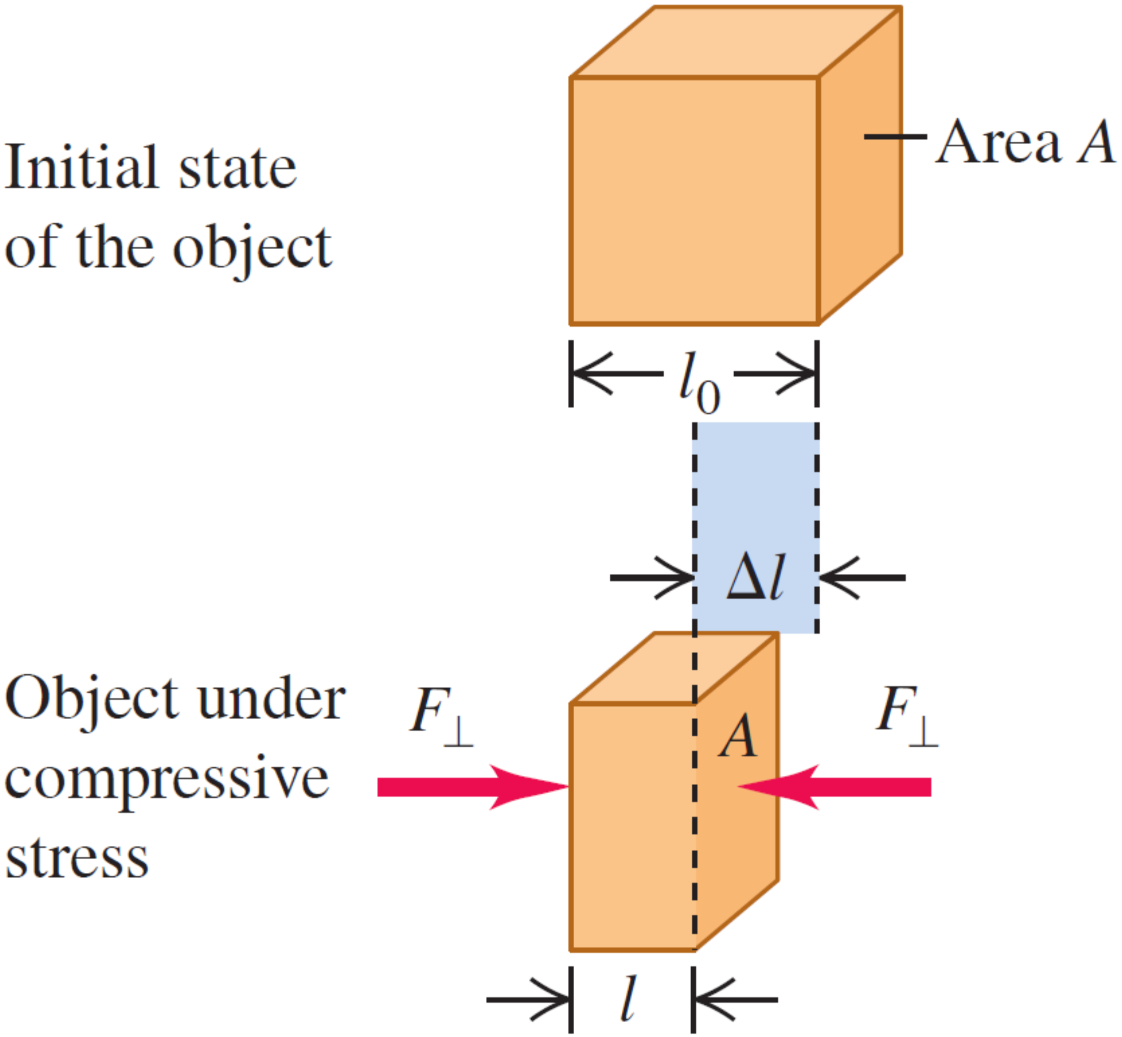
弹性系数，或者劲度系数

常见材料的杨氏模量

TABLE 11.1 Approximate Elastic Moduli

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Silicone rubber	0.001×10^{10}	0.2×10^{10}	0.0002×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}
Tendon (typical)	0.12×10^{10}	—	—

压应力/应变
Compressive stress/strain



$$\text{Compressive stress} = \frac{F_\perp}{A}$$

$$\text{Compressive strain} = \frac{\Delta l}{l_0}$$

与拉应力/应变的区别：符号相反

例8-5

A steel rod 2.0 m long has a cross-sectional area of 0.30 cm^2. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.

b）体应力/应变
Bulk stress/strain

深海的潜水员和鱼

体应力/应变 （Bulk stress/strain）的定义：

物体所受各向同性（isotropic）的应力和应变

物体在流体中所受各向同性压强：

Pressure in a fluid → $p = \dfrac{F_\perp}{A}$

$F_\perp$: Force that fluid applies to surface of an immersed object

A: Area over which force is exerted

体应变： $\text{Bulk (volume) strain} = \dfrac{\Delta V}{V_0}$

体弹性模量（Bulk modulus）：

Bulk modulus for compression → $$B = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V/V_0}$$

Δp: Additional pressure on object

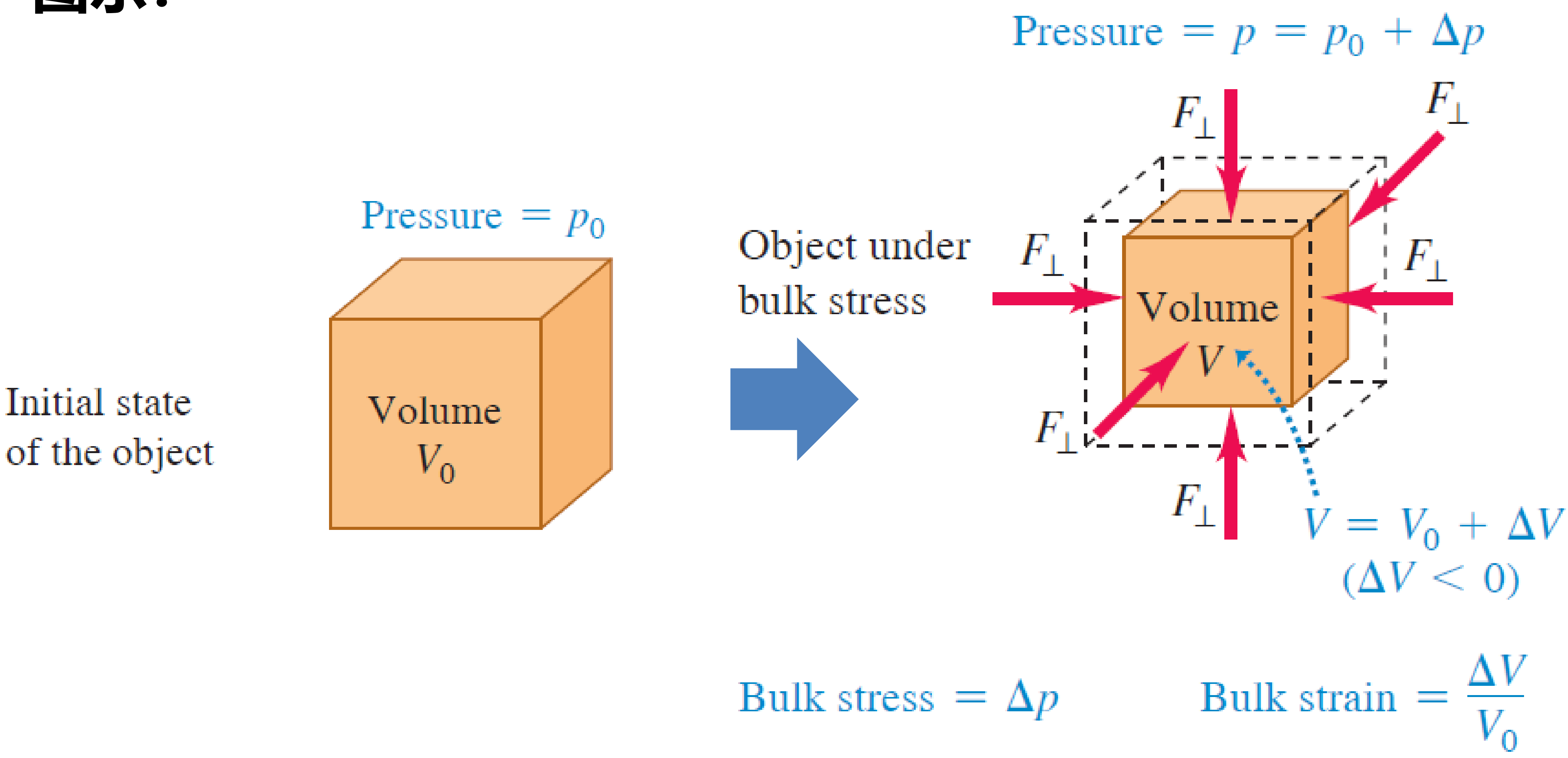
ΔV: Change in volume (see Fig. 11.17)

V_0: Original volume (see Fig. 11.17)

压缩系数（compressibility）：

$$k = \frac{1}{B} = -\frac{\Delta V/V_0}{\Delta p} = -\frac{1}{V_0}\frac{\Delta V}{\Delta p}$$

图示：

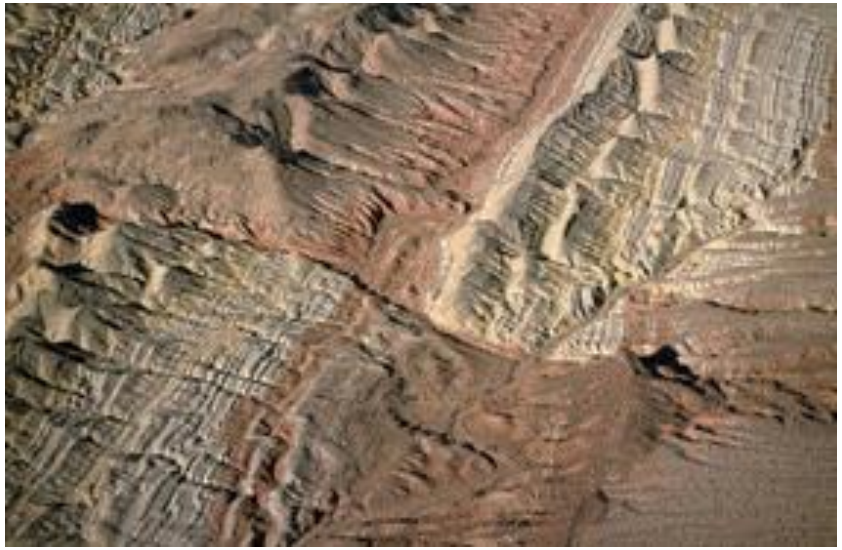


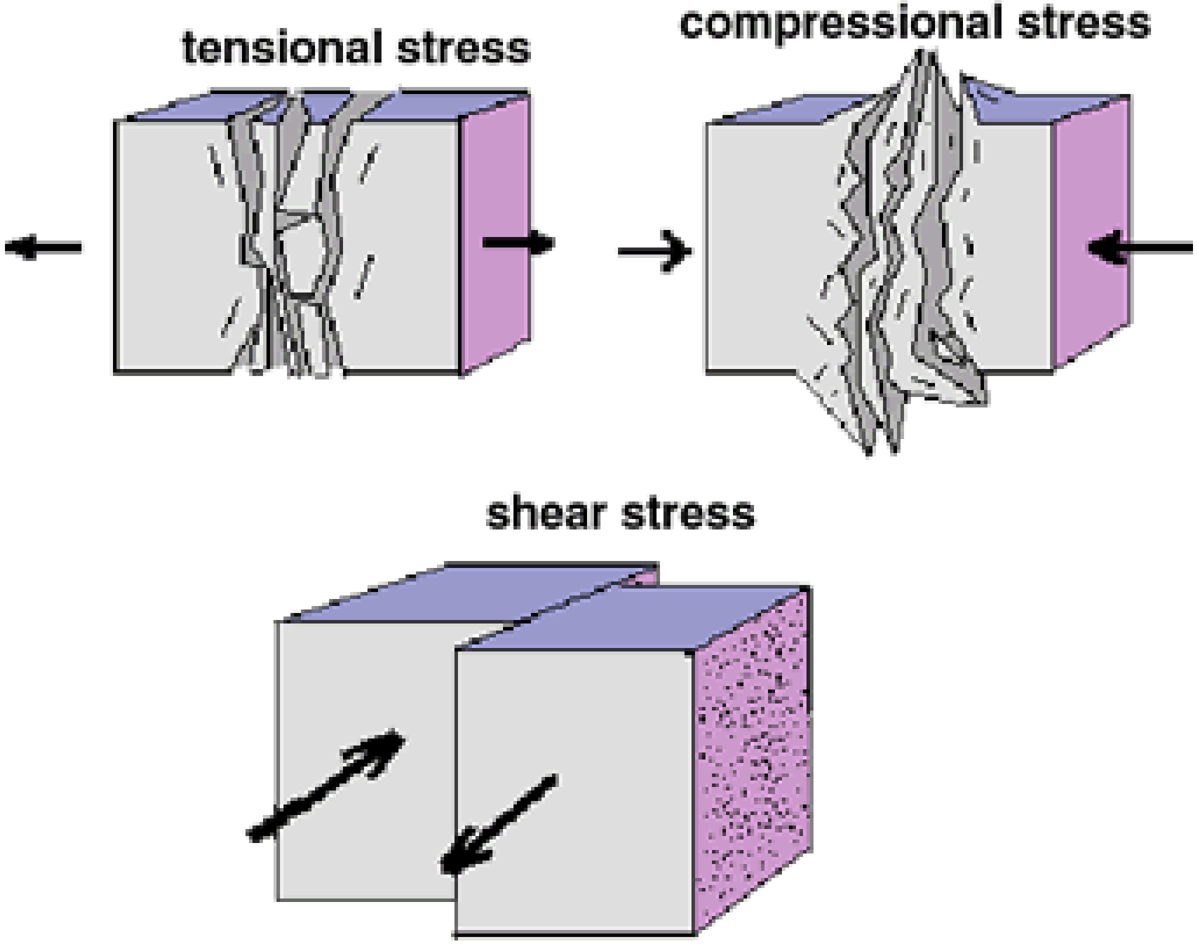
Initial state
of the object
Pressure = p_0
Volume
V_0
Object under
bulk stress
Pressure = $p = p_0 + \Delta p$
$F_\perp$
Volume
V
$V = V_0 + \Delta V$
$(\Delta V < 0)$
Bulk stress = Δp
Bulk strain = $\frac{\Delta V}{V_0}$

例8-6

A hydraulic press contains 0.25 m^3 (250 L) of oil. Find the decrease in the volume of the oil when it is subjected to a pressure increase $\Delta p = 1.6 \times 10^7$ Pa (about 160 atm or 2300 psi). The bulk modulus of the oil is $B = 5.0 \times 10^9$ Pa (about 5.0×10^4 atm), and its compressibility is $k = 1/B = 20 \times 10^{-6}$ atm^{-1}.

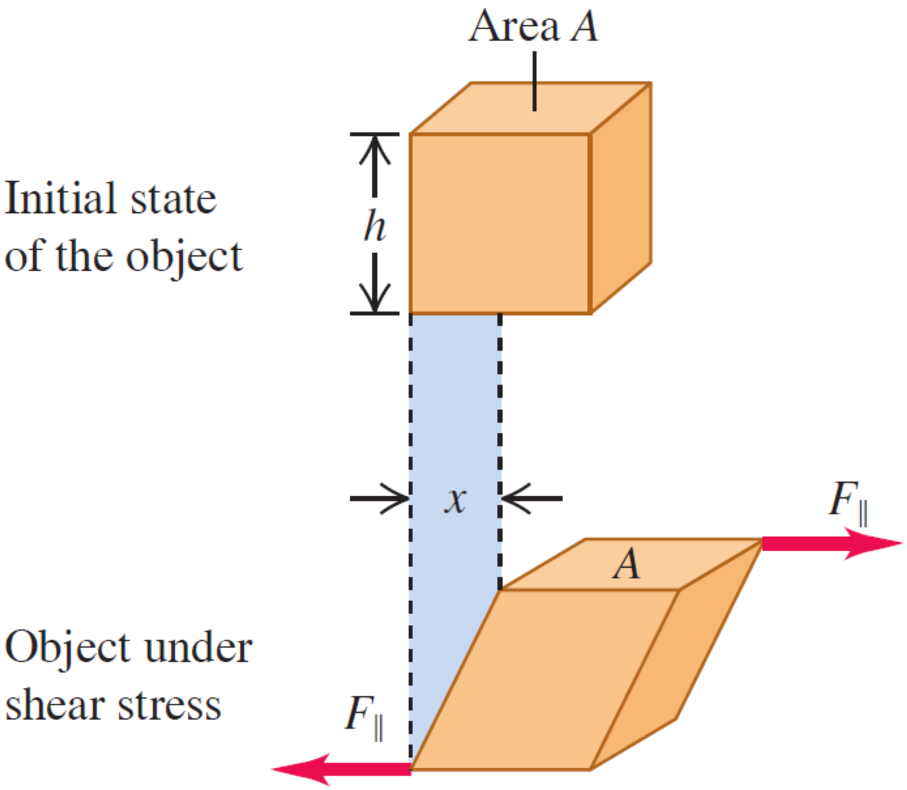
c) 切应力/应变
Shear stress/strain





切应力/应变 Shear stress/strain定义：

Forces of equal magnitude but opposite direction act tangent to the surfaces of opposite ends of the object.



切应力：

$$\text{Shear stress} = \frac{F_{\parallel}}{A}$$

切应变：

$$\text{Shear strain} = \frac{x}{h}$$

切向模量：

Force applied tangent to surface of object

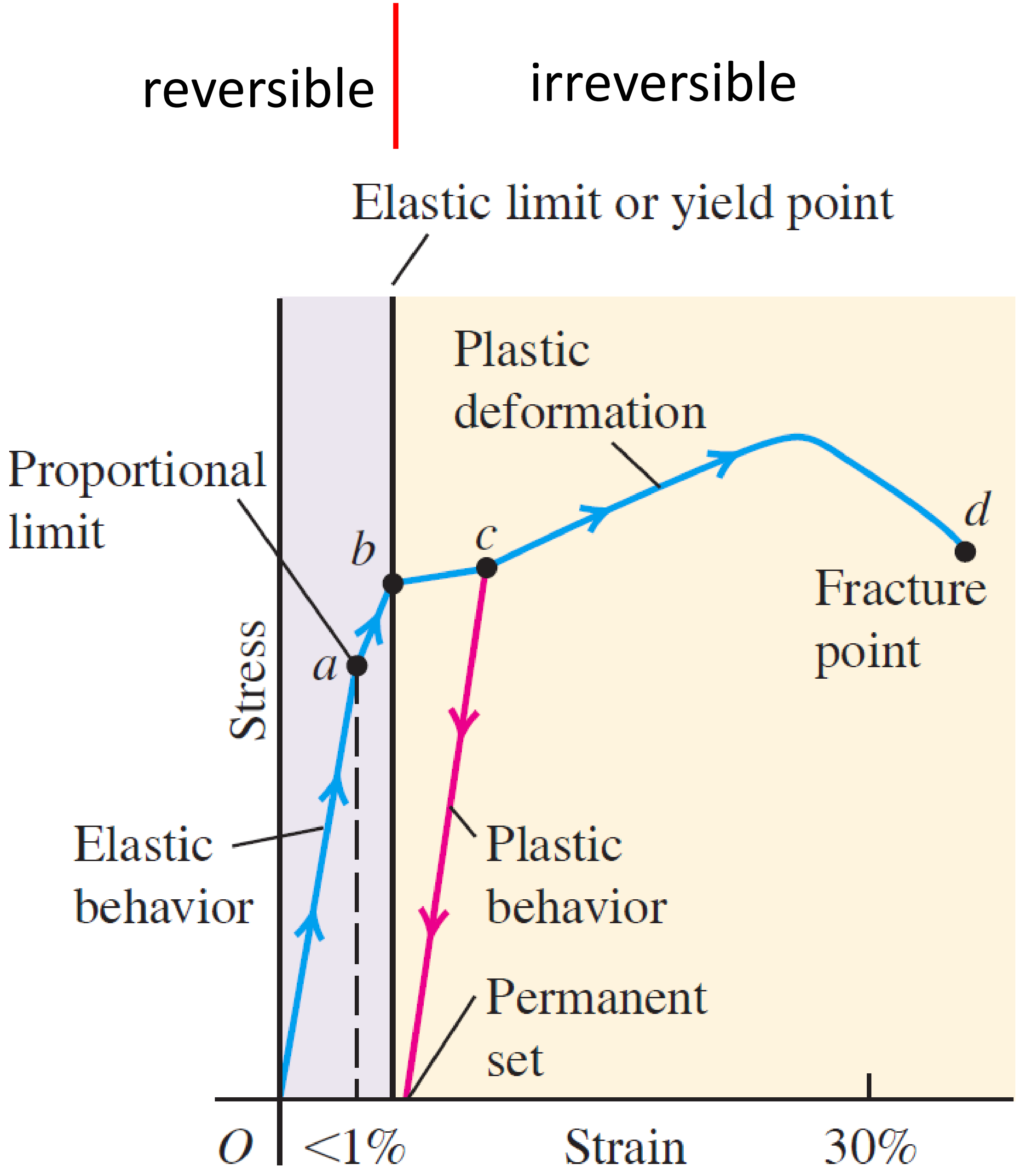
$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A}\frac{h}{x}$$

Area over which force is exerted

d）胡克定律的局限性

胡克定律仅适用于弹性形变区，即应变和应力成线性正比

如右图所示：胡克定律仅适用于小应变区间。



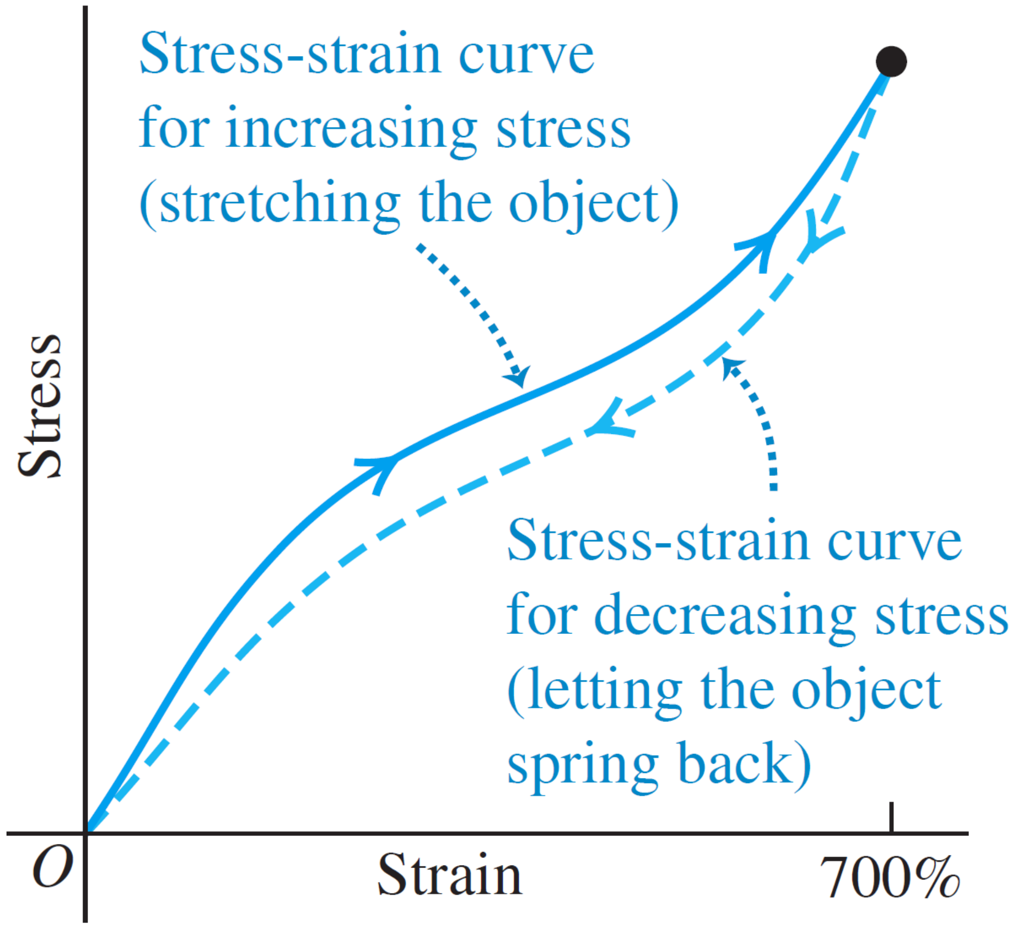
Breaking stress: minimum stress to cause fracture

TABLE 11.3 Approximate Breaking Stresses

Material	Breaking Stress (Pa or N/m^2)
Aluminum	2.2×10^8
Brass	4.7×10^8
Glass	10×10^8
Iron	3.0×10^8
Steel	$5-20 \times 10^8$
Tendon (typical)	1×10^8

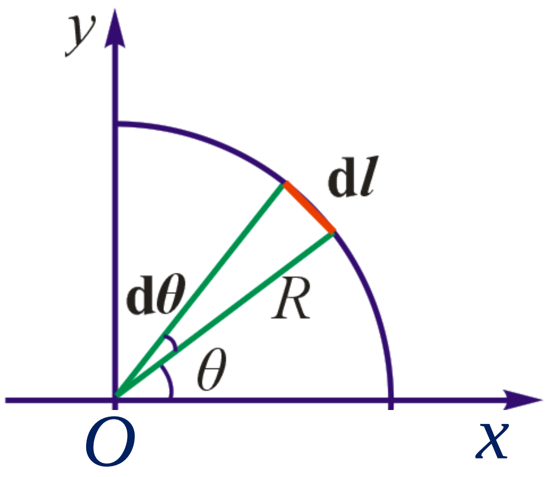
胡克定律不适用的另一情况: 弹性回滞曲线

硫化橡胶
vulcanized rubber



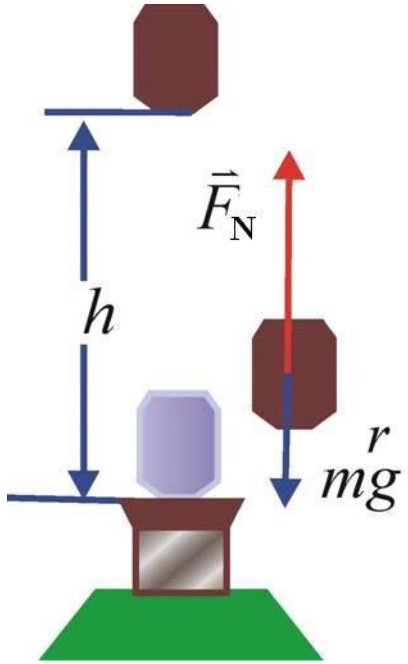
HW #1

一段均匀铁丝弯成1/4圆形，其半径为R，质量为m，求此段铁丝的质心。



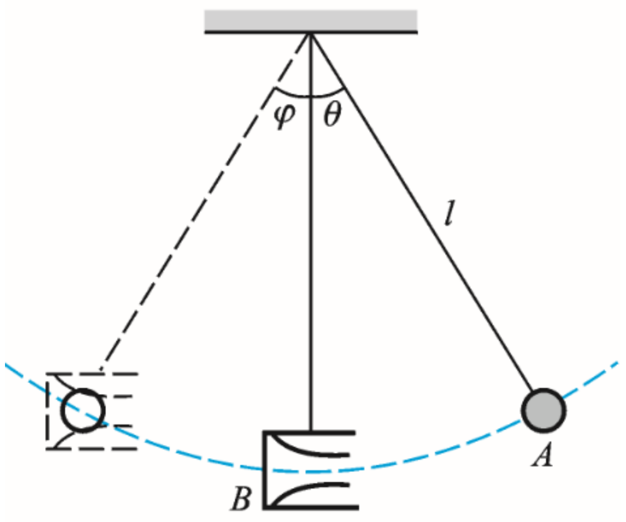
HW #2

质量m =0.5 t的重锤，从高度h =1 m处自由落到受锻压的工件上，工件发生形变。如果作用的时间τ=0.1 s，试求锤对工件的平均冲力。



HW #3

如图，*A*为一小球，*B*为蹄状物，质量分别为m_1和m_2. 开始时，将*A*球从张角θ 处落下，然后与静止的*B*物相碰撞，嵌入*B*中一起运动，求两物到达最高处的张角φ 。



HW #4

一质量为 m的物体,从质量为 M 的圆弧形槽顶端由静止滑下,设圆弧形槽的半径为R,张角为π/2。如所有摩擦都可忽略,求:

(1)物体刚离开槽底端时,物体和槽的速度各是多少?

(2)在物体从A滑到B的过程中,物体对槽所做的功。