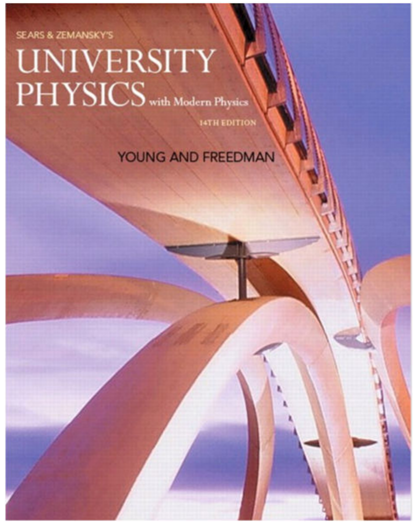
普通物理I PHYS1181.03 第3讲

# 质点动力学 Dynamics



其他重要的坐标系：极坐标系、自然坐标系

运动和力

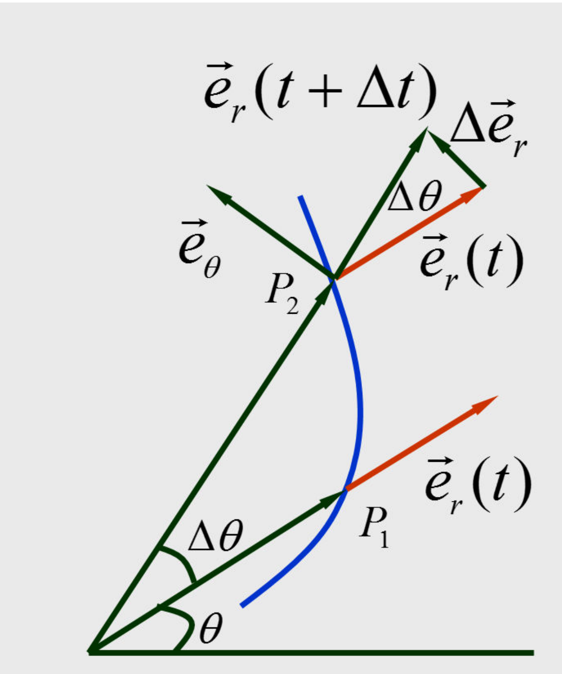
牛顿三大定律

惯性参考系

# 其他重要的坐标系：极坐标系、自然坐标系

# 速度的分量形式：平面极坐标系

$$\vec{r}(t)=r(t)\vec{e}_r(t)$$




$$\vec{v}=\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}(r\vec{e}_r)=\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r+r\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}$$

$\Delta t\to 0$ 时，方向：$\Delta\vec{e}_r // \vec{e}_\theta$

大小：$|\Delta\vec{e}_r|=|\vec{e}_r|\Delta\theta=\Delta\theta \Rightarrow \Delta\vec{e}_r=\Delta\theta\vec{e}_\theta$

$$\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}=\lim_{\Delta t\to 0}\frac{\Delta\vec{e}_r}{\Delta t}=\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}\vec{e}_\theta=\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta$$

$$\vec{v}=\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r+r\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta=v_r\vec{e}_r+v_\theta\vec{e}_\theta$$

# 加速度的分量形式

**直角坐标系**

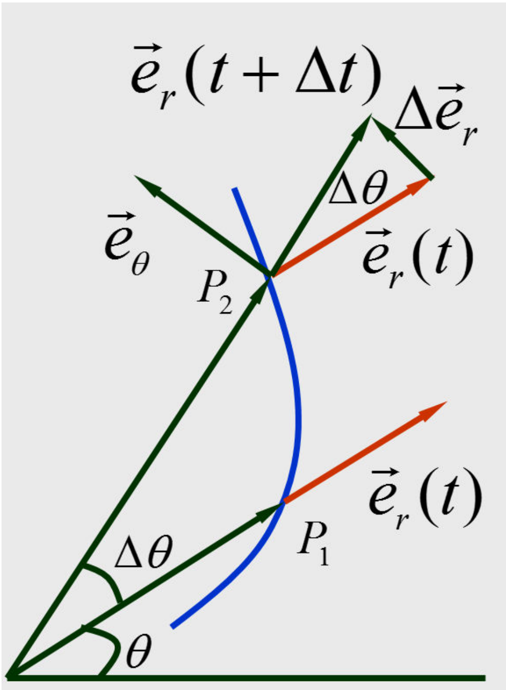
$$\vec{a}=\frac{dv_x}{dt}\hat{\imath}+\frac{dv_y}{dt}\hat{\jmath}+\frac{dv_z}{dt}\hat{k}=\frac{d^2x}{dt^2}\hat{\imath}+\frac{d^2y}{dt^2}\hat{\jmath}+\frac{d^2z}{dt^2}\hat{k}$$

**平面极坐标系**

$$\vec{a}=\frac{d\vec{v}}{dt}=\frac{d(\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta)}{dt}$$

$$=\frac{\mathrm{d}^2r}{\mathrm{d}t^2}\vec{e}_r+\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}+\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta+r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\vec{e}_\theta+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\cdot\frac{\mathrm{d}\vec{e}_\theta}{\mathrm{d}t}$$

# 加速度的分量形式：平面极坐标系




如前所述： $\dfrac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}=\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta$ 　　同理可得： $\dfrac{\mathrm{d}\vec{e}_\theta}{\mathrm{d}t}=-\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_r$

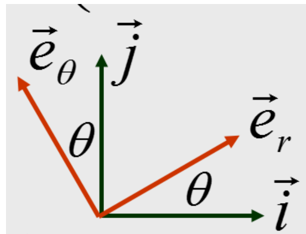
简单的推导：

$$\vec{e}_r=\cos\theta\vec{i}+\sin\theta\vec{j}$$

$$\vec{e}_\theta=-\sin\theta\vec{i}+\cos\theta\vec{j}$$

$$\dot{\vec{e}}_r=\left(-\sin\theta\vec{i}+\cos\theta\vec{j}\right)\dot{\theta}=\dot{\theta}\vec{e}_\theta$$

$$\dot{\vec{e}}_\theta=\left(-\cos\theta\vec{i}-\sin\theta\vec{j}\right)\dot{\theta}=-\dot{\theta}\vec{e}_r$$



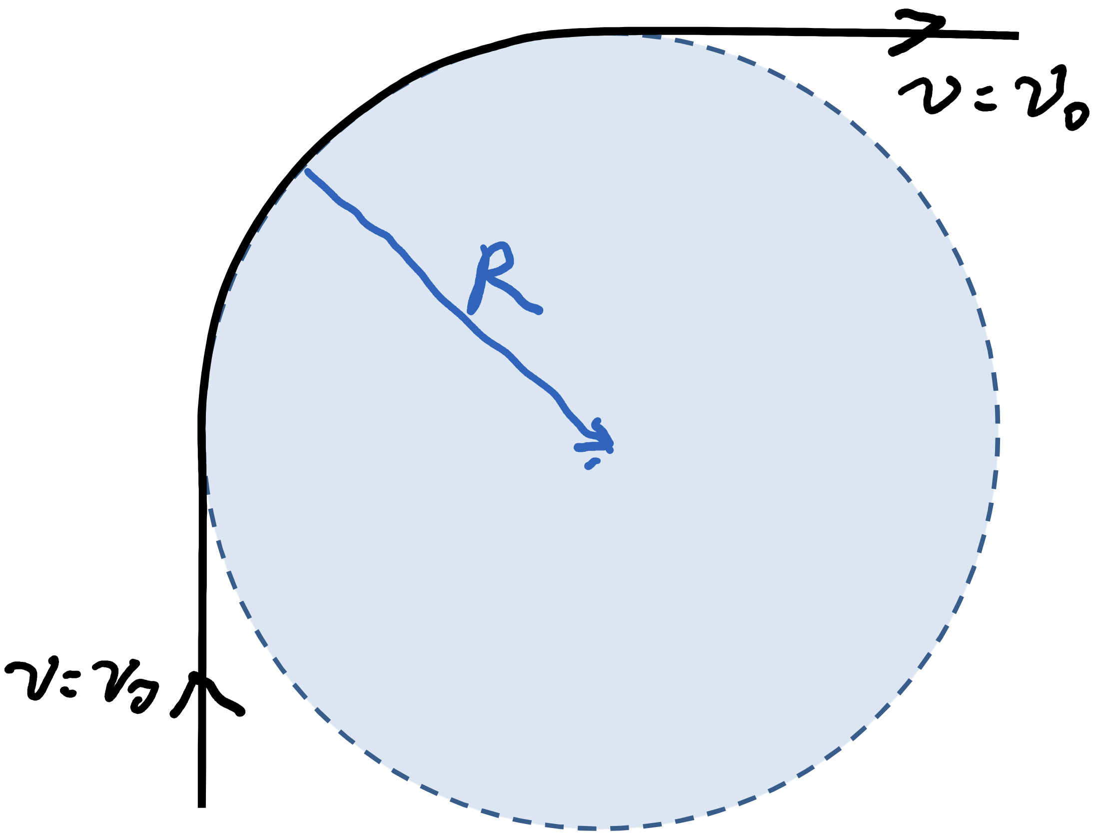
# 加速度的分量形式：平面极坐标系

$$\vec{a}=\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\vec{e}_r+\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta+\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta+r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\vec{e}_\theta-r\frac{\mathrm{d}\theta}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_r$$

$$=\left(\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}-r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right)\vec{e}_r+\left(r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+2\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)\vec{e}_\theta$$

$$\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\vec{e}_r+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

# 例子：匀速率过弯




$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

$$\vec{a} = -r\dot{\theta}^2\vec{e}_r + r\ddot{\theta}\vec{e}_\theta$$

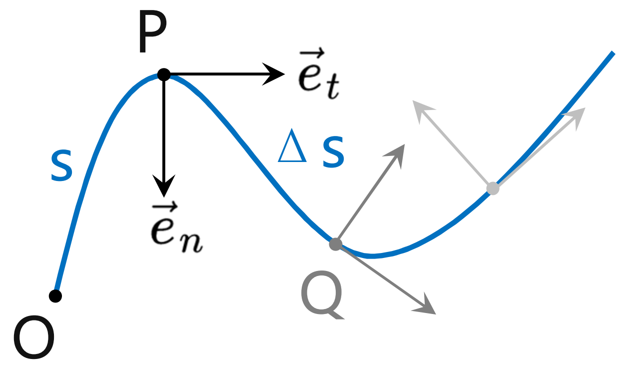
$$r\dot{\theta}^2 = v^2 / r$$

$$r\ddot{\theta} = 0$$

# 自然坐标系

在某质点运动的轨迹线上任取一点O为自然坐标原点，以质点所在位置P点与O点间轨迹的路程 $s$ 来确定质点的位置，则称$s$ 为质点的自然坐标，其运动方程：

$$s = s(t)$$




当质点经Δt时间内从P点达Q点，运动的路程为$\Delta s$：

$$\Delta s = s(t + \Delta t) - s(t)$$

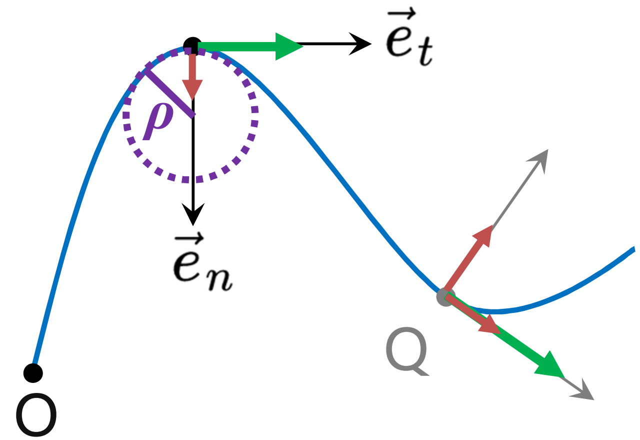
在质点轨迹切向取一单位矢量$e_t$，其称之为**切向单位矢量** $\vec{e}_t$

在$e_t$垂直且指向质点轨迹凹侧的方向处取单位矢量$e_n$，称作**法向单位矢量** $\vec{e}_n$

速度矢量的大小（速率）： $v=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\frac{ds}{dt}$

速度矢量的方向总是沿着为轨迹的切向，表示为： $\vec{v}=\frac{ds}{dt}\vec{e}_t$

加速度矢量： $\vec{a}=a_1\vec{e}_t+a_2\vec{e}_n$



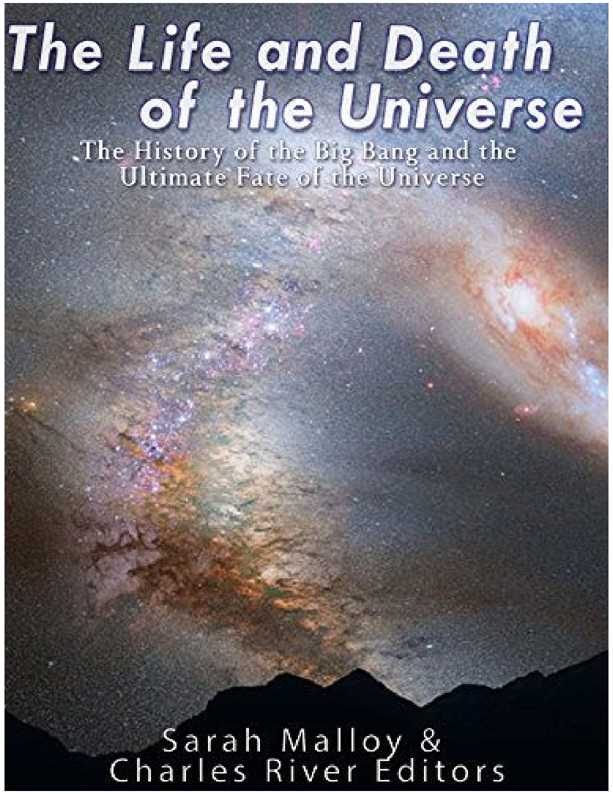

$a_1$:切向加速度，改变速度的大小

$a_2$:法向加速度，改变速度的方向

$$a_1=\frac{dv}{dt}\qquad a_2=v\frac{d\theta}{dt}=v\frac{ds}{dt}\frac{d\theta}{ds}=\frac{v^2}{\rho}$$

其中，$\rho=\frac{ds}{d\theta}=\left|\frac{ds}{d\theta}\right|$ 为质点处轨迹的曲率半径，始终为正

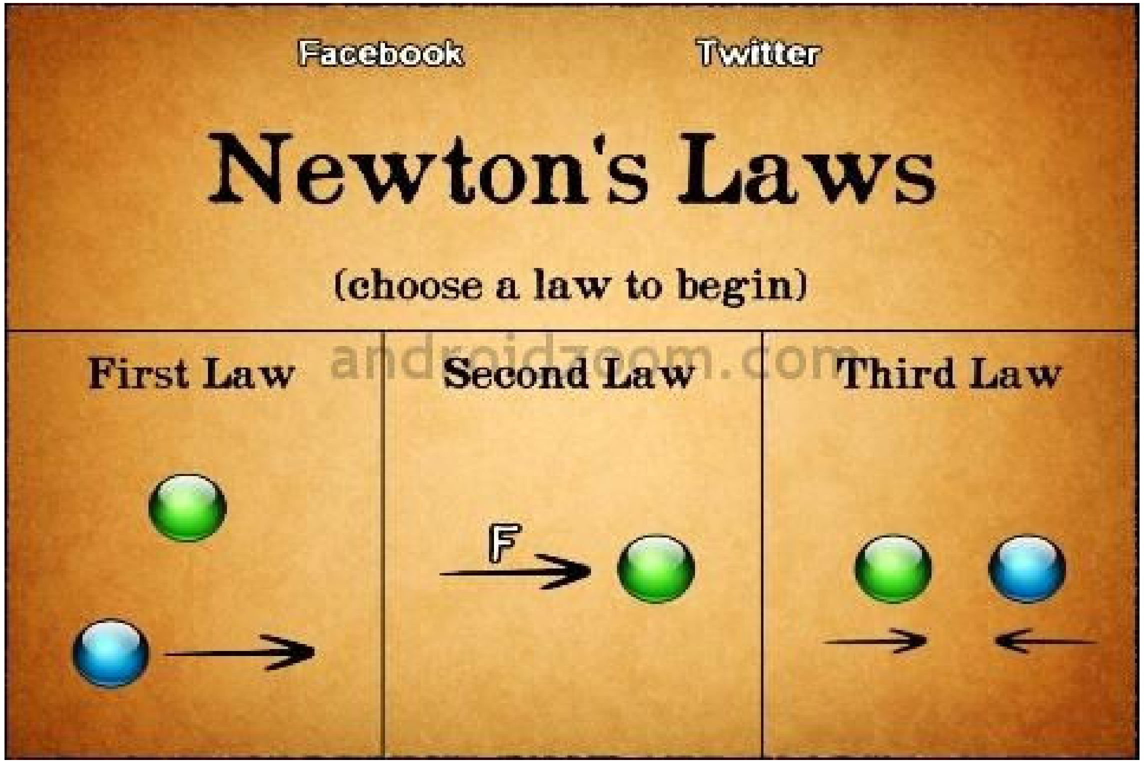
# 运动-力-相互作用




我们的宇宙生于相互作用。

没有相互作用，等于死寂。

# 牛顿三大运动定律




你不推，我不变　　你一推，我就动　　你动我，我动你

1. Every body persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

2. The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

3. To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

# 基本的概念

动力学，描述运动的“原因”，即受力与运动之间的相互关系。

- Newton’s laws of motion: 牛顿运动定律
- Force: 力
- Inertial frame of reference: 惯性参考系

# 力与运动关系的探索

亚里士多德：力是物体运动的原因
伽利略：力是改变物体运动的原因


伽利略：力不是维持运动的原因。

↓

笛卡尔：无外力，等速直线运动

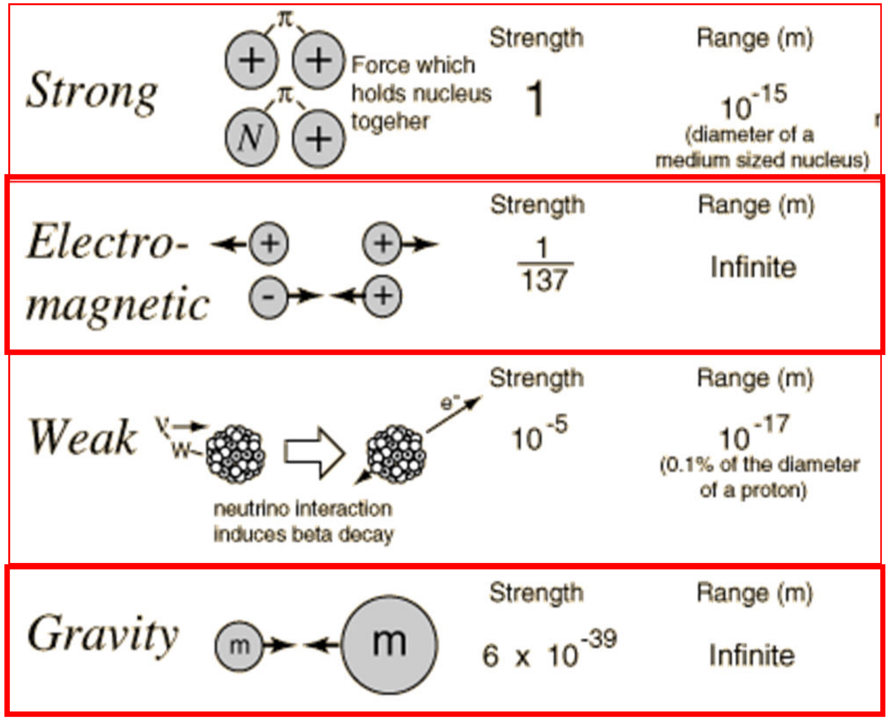
↓

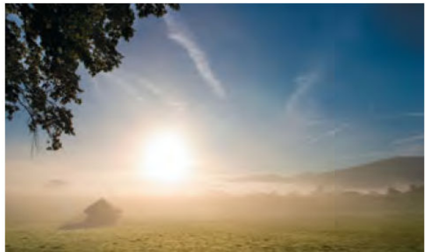
牛顿：无外力，匀速直线运动或静止；直到有外力改变这个状态。

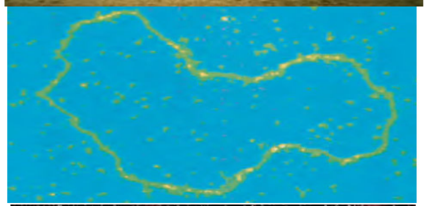
# 力：Force

描述物体之间的相互作用的量。
兼有大小和方向两个属性， 因此为矢量。

四种基本的相互作用





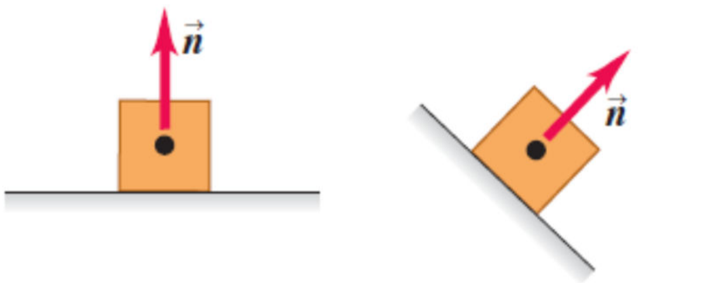

# 力都有多大？

**TABLE 4.1** Typical Force Magnitudes

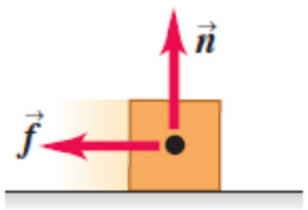
| | |
|---|---|
| Sun's gravitational force on the earth | $3.5 \times 10^{22}$ N |
| Weight of a large blue whale | $1.9 \times 10^{6}$ N |
| Maximum pulling force of a locomotive | $8.9 \times 10^{5}$ N |
| Weight of a 250 lb linebacker | $1.1 \times 10^{3}$ N |
| Weight of a medium apple | 1 N |
| Weight of the smallest insect eggs | $2 \times 10^{-6}$ N |
| Electric attraction between the proton and the electron in a hydrogen atom | $8.2 \times 10^{-8}$ N |
| Weight of a very small bacterium | $1 \times 10^{-18}$ N |
| Weight of a hydrogen atom | $1.6 \times 10^{-26}$ N |
| Weight of an electron | $8.9 \times 10^{-30}$ N |
| Gravitational attraction between the proton and the electron in a hydrogen atom | $3.6 \times 10^{-47}$ N |

# 四种常见的力 Four common types of force

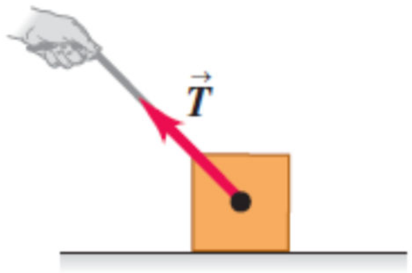
(a) **Normal force $\vec{n}$:** When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



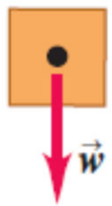

(b) **Friction force $\vec{f}$:** In addition to the normal force, a surface may exert a friction force on an object, directed parallel to the surface.




(c) **Tension force $\vec{T}$:** A pulling force exerted on an object by a rope, cord, etc.




(d) **Weight $\vec{w}$:** The pull of gravity on an object is a long-range force (a force that acts over a distance).

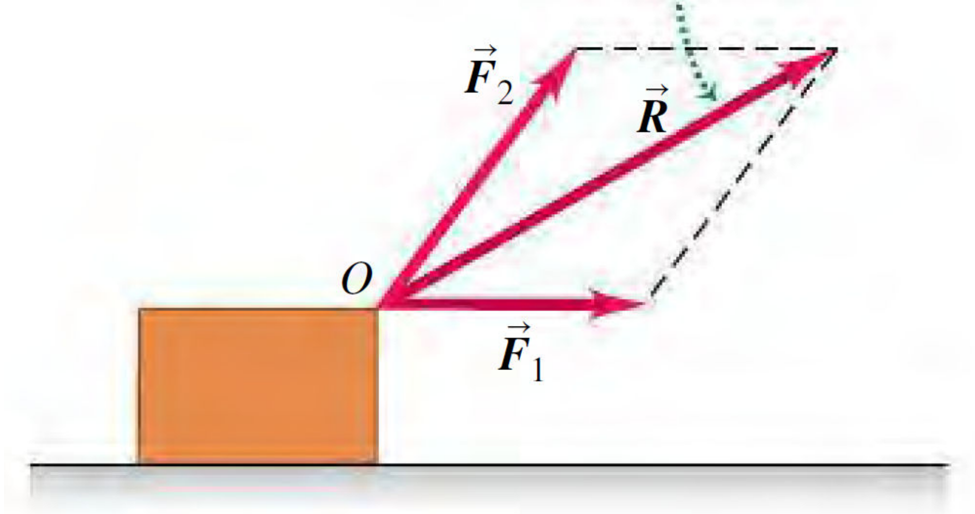


# 合力 Superposition of forces

The net force acting on an object ... $\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$

... is the vector sum, or resultant, of all individual forces acting on that object.

Two forces $\vec{F}_1$ and $\vec{F}_2$ acting on a body at point $O$ have the same effect as a single force $\vec{R}$ equal to their vector sum.




2D:

$$R_x = \sum F_x \qquad R_y = \sum F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

3D:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

# 牛顿第一定律（惯性定律）

**NEWTON'S FIRST LAW OF MOTION** An object acted on by no net external force has a constant velocity (which may be zero) and zero acceleration.

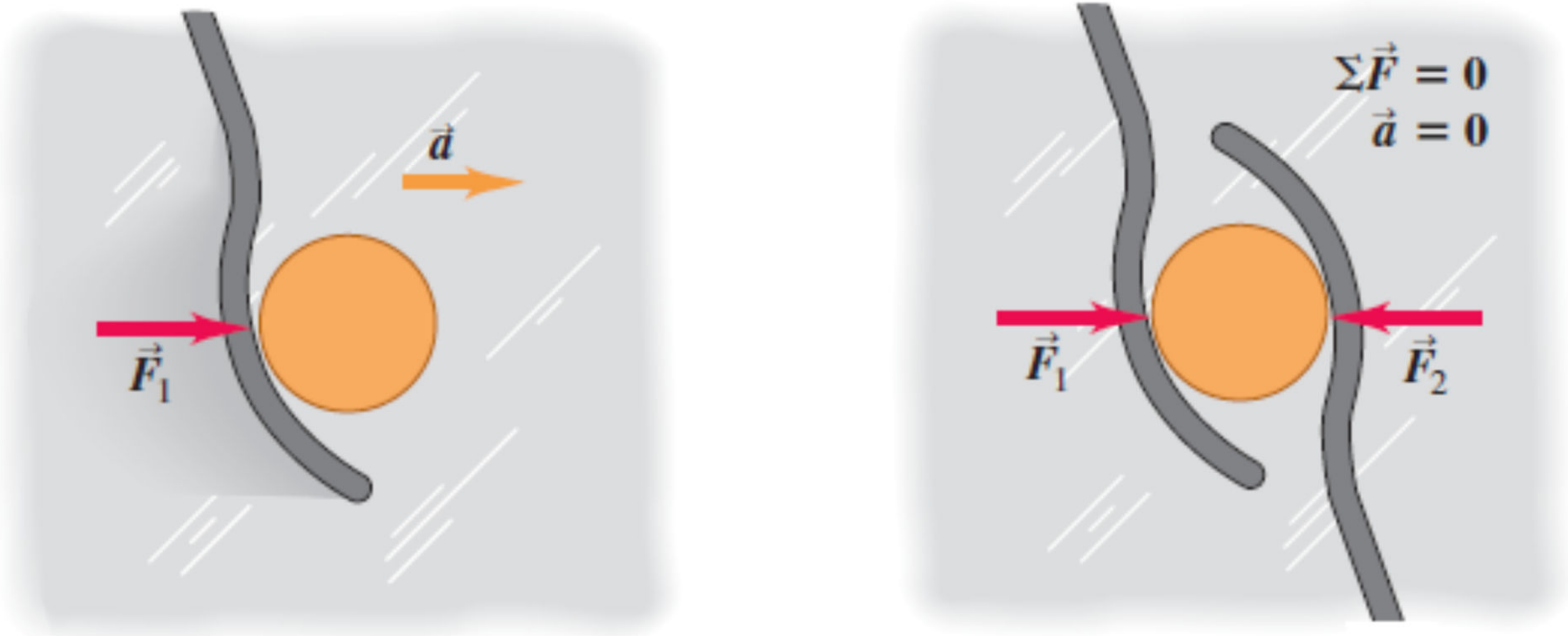
任何物体如果没有外力的作用，都将保持静止或作匀速直线运动的状态。

现代版本的描述：

自由粒子永远保持静止或匀速直线运动的状态。

# 牛顿第一定律的数学表达：

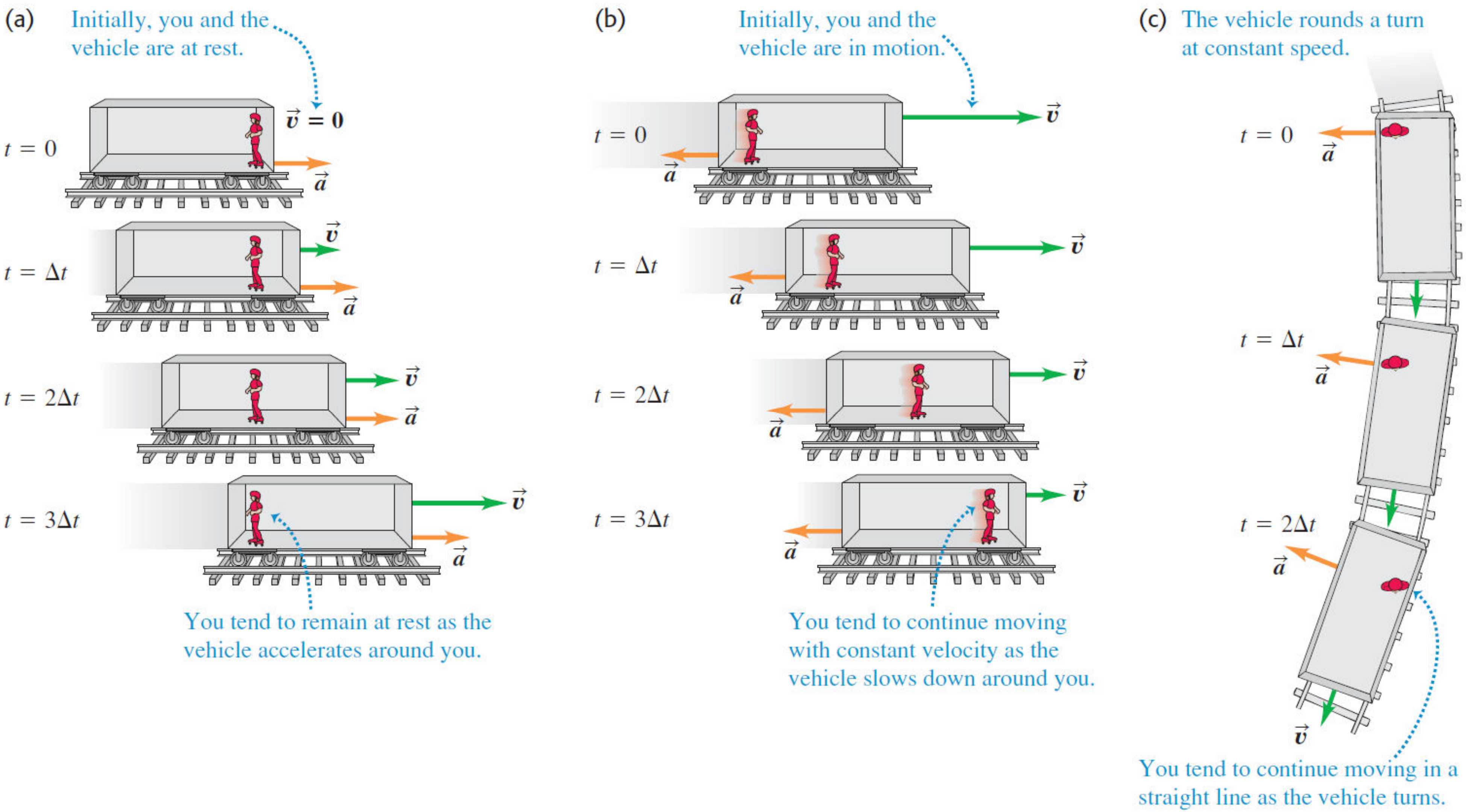
**Newton's first law:**
Net external force on an object ... $\sum \vec{F} = 0$ ... must be zero if the object is in **equilibrium**.




注意：牛顿第一定律特指外力 External force

近似匀速直线运动：冰球 Hockey puck（无阻力）

# 牛顿第一定律定义惯性参考系（inertial frame of reference）



# 牛顿第一定律的意义

**(1)** 定义了惯性参考系 ← 在牛顿第一定律适用的参考系才是惯性参考系

物体静止或匀速直线运动，相对哪个参照系?

——惯性参考系

**(2)** 定义了物体的惯性和力的概念

- 物体保持运动状态的特性——惯性 （**inertia**）
- 改变物体运动状态的原因——力 （物体间的相互作用）

# 惯性参考系：inertial frame of reference

我们接触到的惯性参考系：地球表面（近似）

我们接触到的非惯性参考系：加速减速的车厢，过弯道的火车，启动停止的电梯等

# 惯性参考系：inertial frame of reference

惯性坐标系不唯一： 如果参考系B是惯性坐标系，坐标系A相对于坐标系B的速度是恒定的，那么B也是惯性坐标系。

常量 constant

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

相对于惯性参考系静止或者匀速直线运动的参考系，均为惯性参考系。

因此，惯性定律中描述的静止和匀速直线运动，并没有差别。

## Test your understanding of ‘Inertial Frame of Reference’

In which of the following situations is there zero net external force on the object?

1. an airplane flying due north at a steady 120 m/s and at a constant altitude,
2. a car driving straight up a hill with a 3° slope at a constant 90 km/h;
3. a hawk circling at a constant 20 km/h at a constant height of 15 m above an open field:
4. a box with slick, frictionless surfaces in the back of a truck as the truck accelerates forward on a level road at 5 m/s?

## Improve your understanding of 'Inertial Frame of Reference'

The radius of the earth's orbit around the sun (assumed to be circular) is $1.50 \times 10^8$ km, and the earth travels around this orbit in 365 days.

a) What is the magnitude of the orbital velocity of the earth, in m/s?
b) What is the magnitude of the radial acceleration of the earth toward the sun, in $m/s^2$?

# 牛顿第二定律:

**NEWTON'S SECOND LAW OF MOTION** **If a net external force acts on an object, the object accelerates. The direction of acceleration is the same as the direction of the net external force. The mass of the object times the acceleration vector of the object equals the net external force vector.**

**Newton's second law:** If there is a net external force on an object ...

$$\sum \vec{F} = m\vec{a}$$

Mass of object

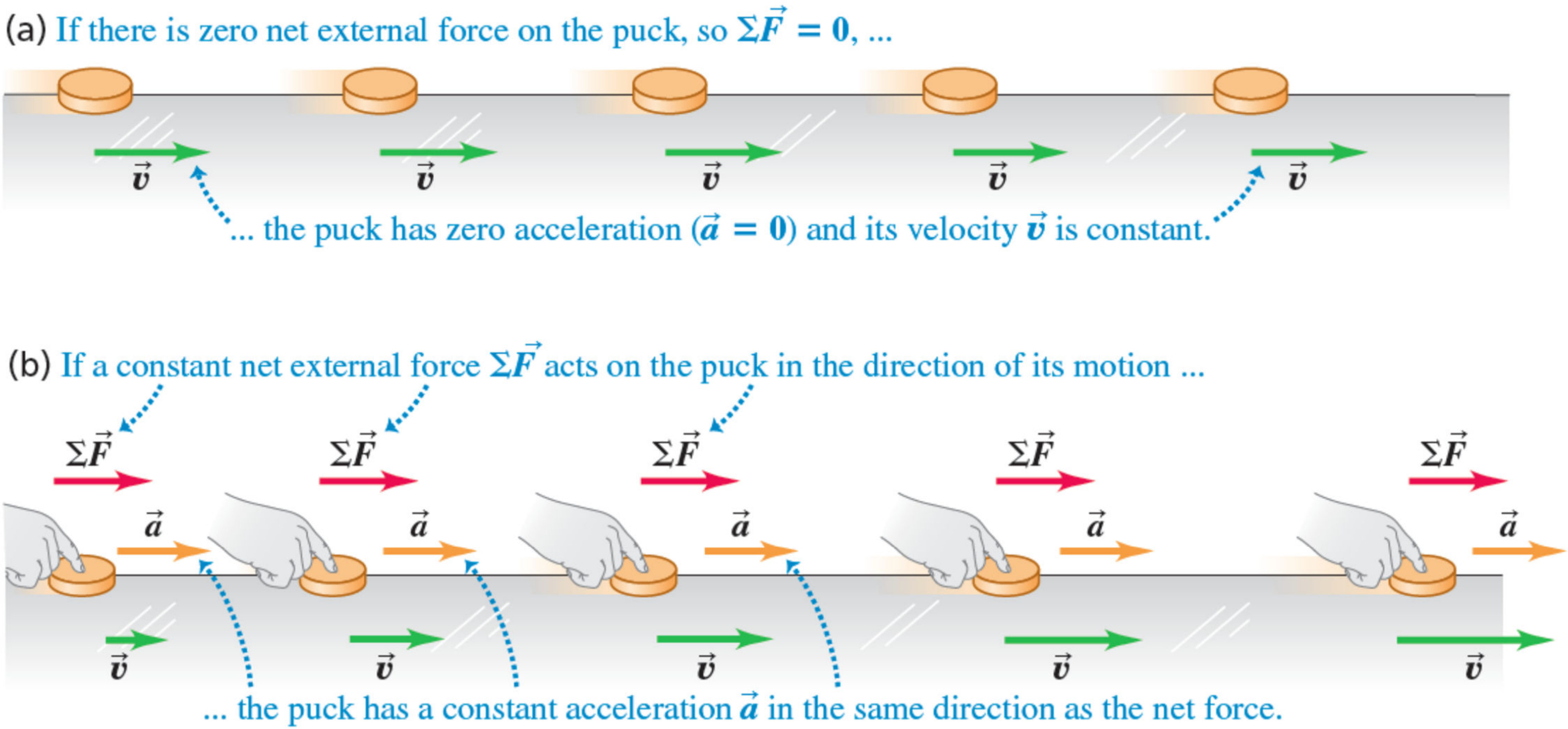
... the object accelerates in the same direction as the net external force.

在外力作用下，质点的加速度方向 $\vec{a}$ 与外力方向相同，大小等于外力除以质量。

# 例子：冰球 Hockey puck on ice

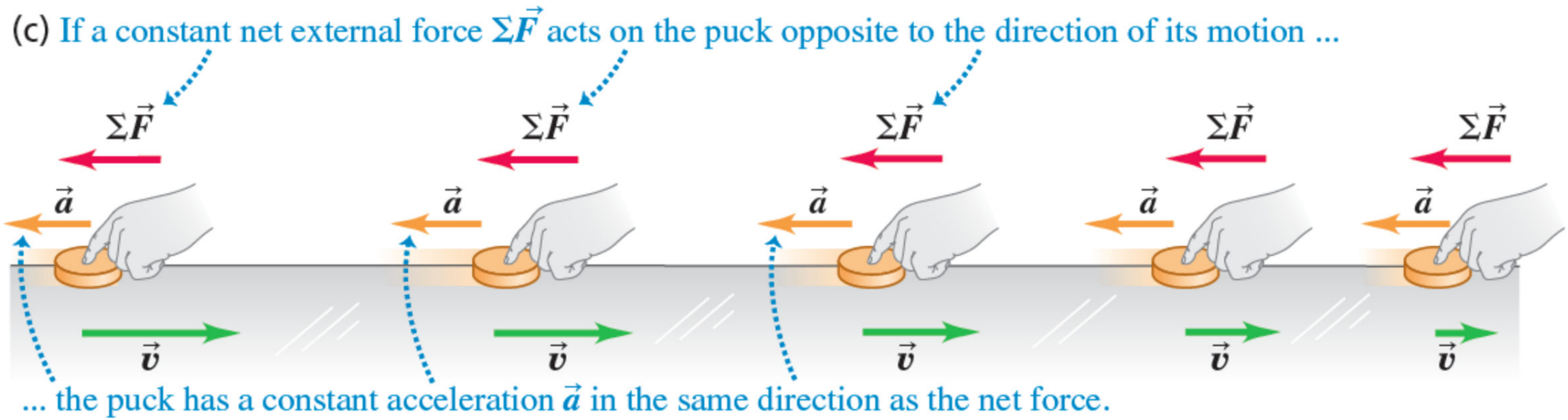
匀速

加速



# 例子：冰球 Hockey puck on ice

减速



# 牛顿第二定律

**第二定律同时定义（a）力的量度**

**（b）物体（惯性）质量**

**力的量度和物体质量通过第二定律协调定义。**

**在国际单位制中：**$\vec{F}$，N；$m$，kg；$a$，m/s$^2$

根据牛顿第二定律定义 $\Rightarrow \vec{a} = \dfrac{\sum_i \vec{F}_i}{m}$

你们熟悉的F=ma, 矢量的形式

# 力、质量、加速度的常用单位

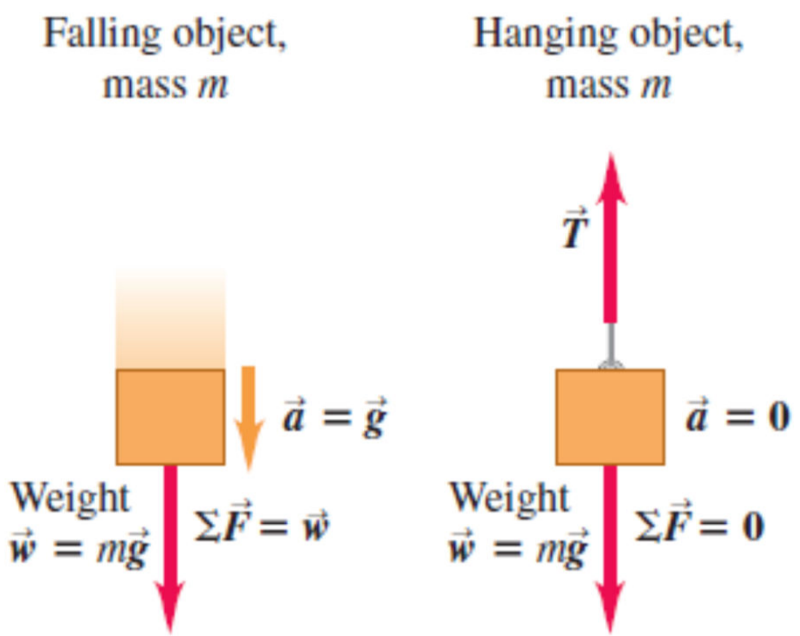
**TABLE 4.2** Units of Force, Mass, and Acceleration

| System of Units | Force | Mass | Acceleration |
|---|---|---|---|
| SI | newton (N) | kilogram (kg) | $m/s^2$ |
| cgs | dyne (dyn) | gram (g) | $cm/s^2$ |
| British | pound (lb) | slug | $ft/s^2$ |

# 重量（w）和质量（m）

$$w = mg$$

- $w$: Magnitude of weight of an object
- $m$: Mass of object
- $g$: Magnitude of acceleration due to gravity




- The relationship of mass to weight: $\vec{w} = m\vec{g}$.
- This relationship is the same whether an object is falling or stationary.

重量是质量在重力作用下的力

同样质量的物体，在地球和月球上具有不同的重量

# 牛顿第二定律的特征

- **牛顿第二定律的瞬时性**

**质点的加速度与其所受的力同时出现或同时消失！**

（然而力的传递需要时间）

- **牛顿第二定律的矢量性**

**在直角坐标系中：** $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

$= m\vec{a}$

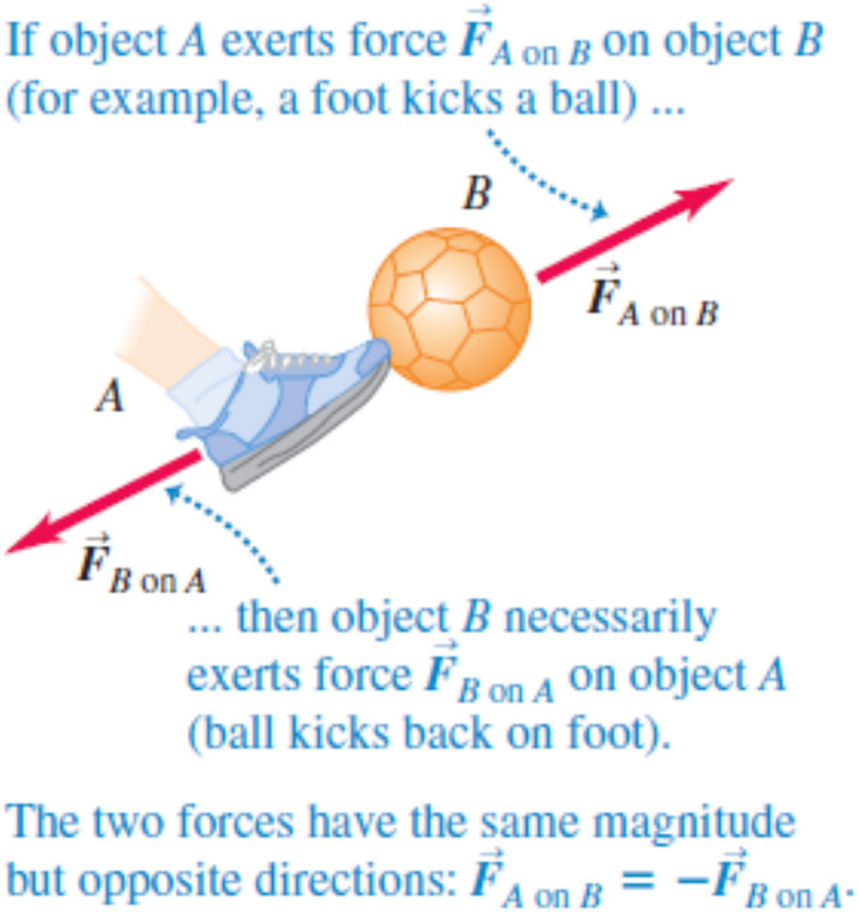
$= ma_x\vec{i} + ma_y\vec{j} + ma_z\vec{k}$

→

$$F_x = ma_x = m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \quad F_y = ma_y = m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$$

$$F_z = ma_z = m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}$$

# 牛顿第三定律（作用力与反作用力定律）




**NEWTON's THIRD LAW OF MOTION: If object A exerts a force on object B (an "action"), then object B exerts a force on object A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on different objects.**

A作用于B的力大小，等于B作用于A的力大小，但方向相反，且两个力作用在不同物体上。

# 牛顿第三定律的特点

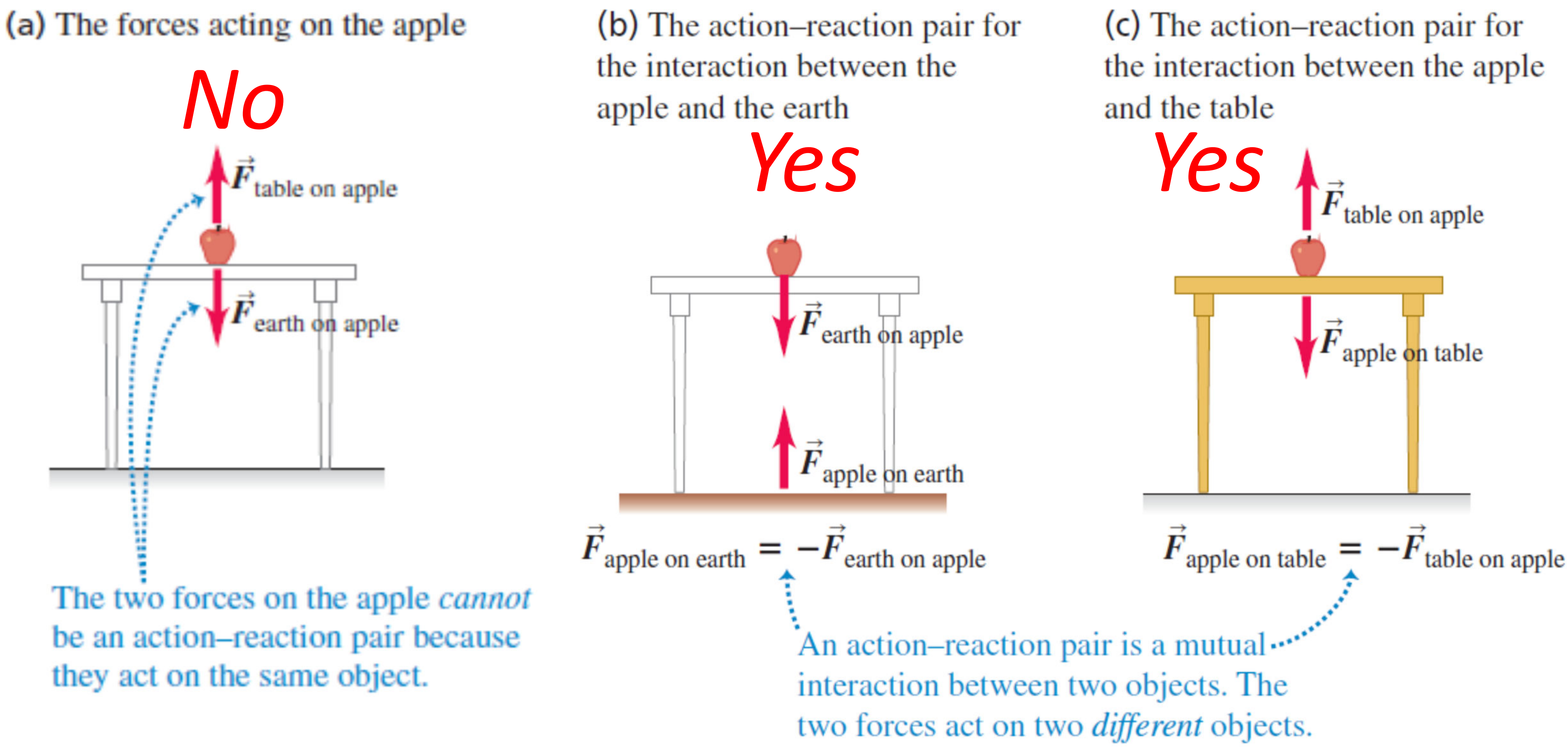
**牛顿第三定律定义了相互作用的性质！**

**除作用在不同物体上以外**

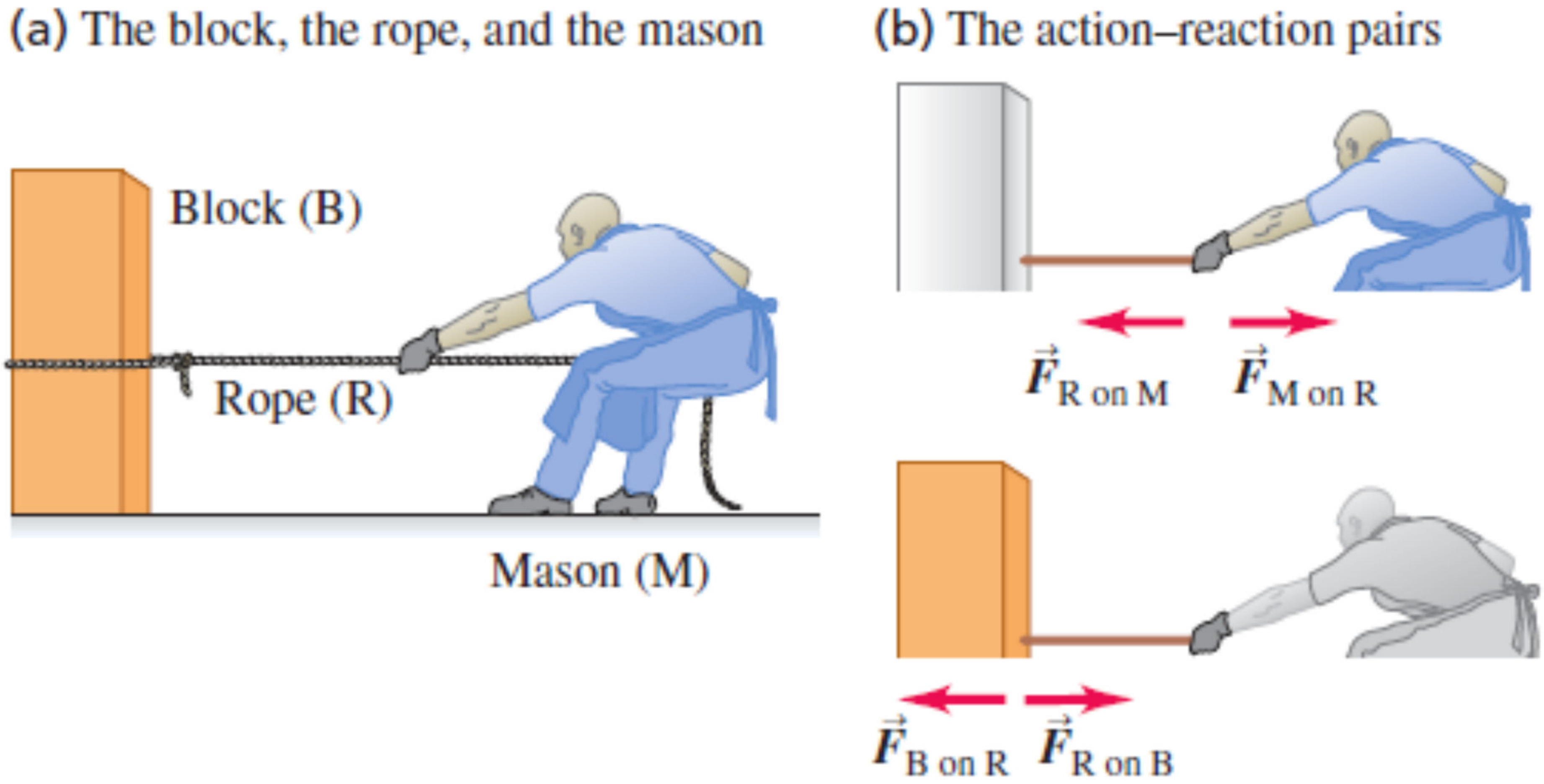
→ **作用力与反作用力本质相同**

* 注意作用力和反作用力与平衡力的差别。

# 作用力和反作用力举例

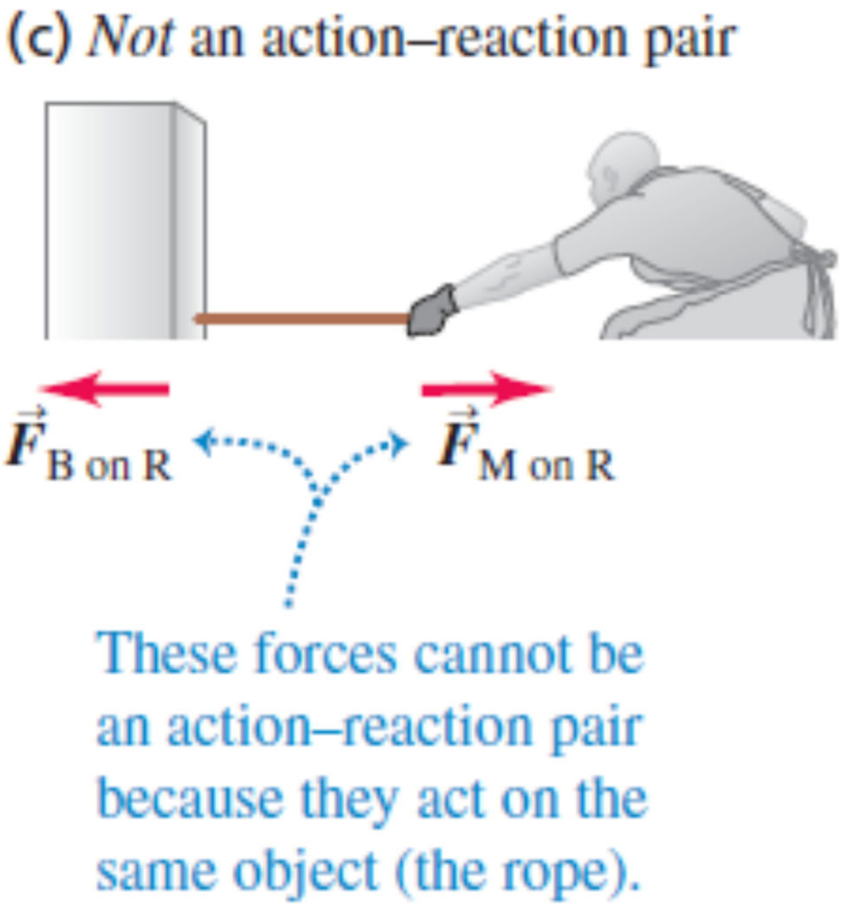

(a) The forces acting on the apple
No
$\vec{F}_{\text{table on apple}}$
$\vec{F}_{\text{earth on apple}}$
The two forces on the apple *cannot* be an action–reaction pair because they act on the same object.
(b) The action–reaction pair for the interaction between the apple and the earth
Yes
$\vec{F}_{\text{earth on apple}}$
$\vec{F}_{\text{apple on earth}}$
$\vec{F}_{\text{apple on earth}} = -\vec{F}_{\text{earth on apple}}$
(c) The action–reaction pair for the interaction between the apple and the table
Yes
$\vec{F}_{\text{table on apple}}$
$\vec{F}_{\text{apple on table}}$
$\vec{F}_{\text{apple on table}} = -\vec{F}_{\text{table on apple}}$
An action–reaction pair is a mutual interaction between two objects. The two forces act on two *different* objects.

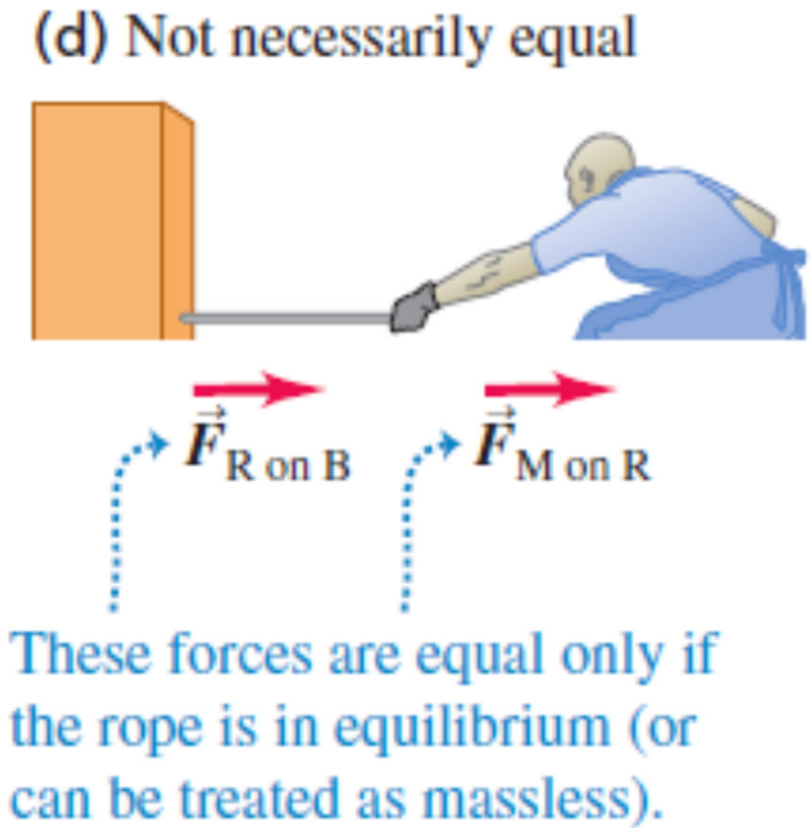
# 作用力和反作用力举例



(a) The block, the rope, and the mason
Block (B)
Rope (R)
Mason (M)
(b) The action–reaction pairs
$\vec{F}_{\text{R on M}}$
$\vec{F}_{\text{M on R}}$
$\vec{F}_{\text{B on R}}$
$\vec{F}_{\text{R on B}}$

# 作用力和反作用力举例




试想下，如果绳子有弹性，力还相等吗？

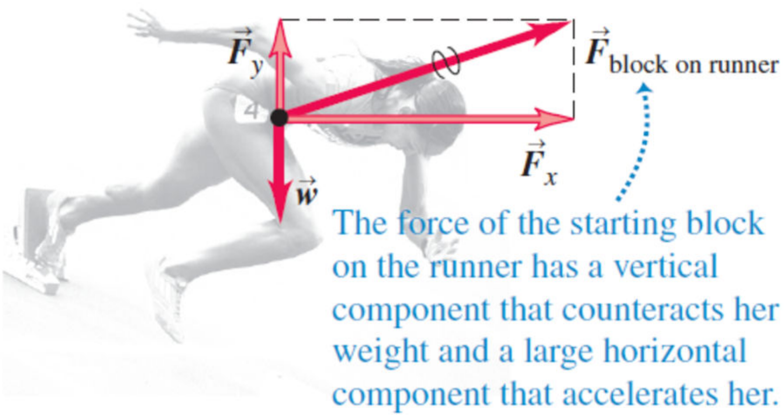
# 自由体受力图 Free-body diagrams

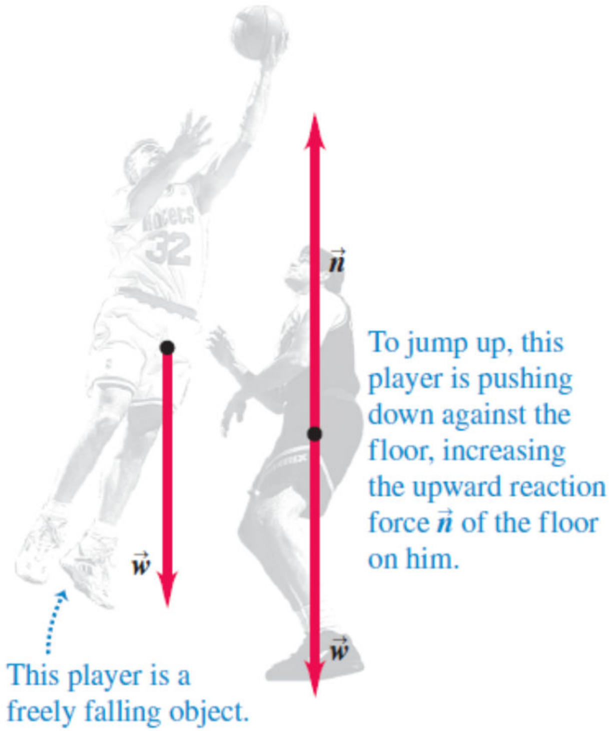
1. Newton’s first and second laws apply to a specific object.

2. Only forces acting on the object matter.

3. Free-body diagrams are essential to help identify the relevant forces. A free-body diagram shows the chosen object by itself, “free” of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces that act on the object.

# 自由体受力图 Free-body diagrams



**TEST YOUR UNDERSTANDING OF SECTION 4.4** Suppose an astronaut landed on a planet where $g = 19.6\ \text{m/s}^2$. Compared to earth, would it be easier, harder, or just as easy for her to walk around? Would it be easier, harder, or just as easy for her to catch a ball that is moving horizontally at 12 m/s? (Assume that the astronaut's spacesuit is a lightweight model that doesn't impede her movements in any way.)

# 例题1：

用力推水平地面上一质量为M的木箱。设力F与水平面的夹角为θ，木箱与地面之间的滑动摩擦系数和静摩擦系数分别为$\mu_k$和$\mu_s$。

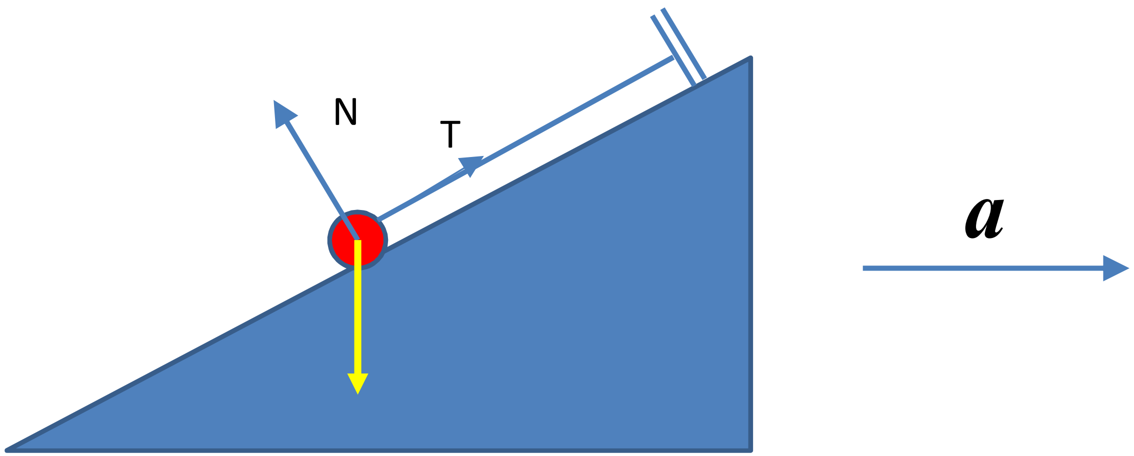
（1）要推动木箱，F至少要多大？此后木箱匀速前进，F应需多大？
（2）证明当θ大于某一角度时，无论用多大的力也不能推动木箱。此θ角是多大？

# 例题2：

设质量m=0.5 kg的小球挂在倾角为30度的光滑斜面上。

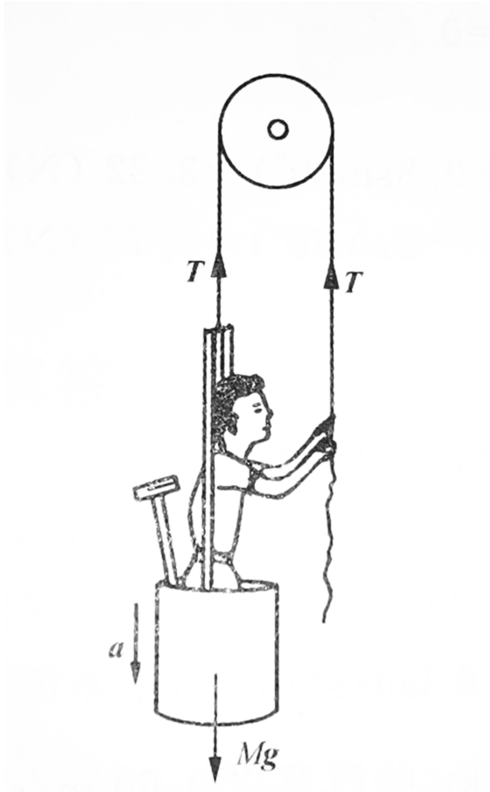
（1）当斜面以加速度a=2 $m/s^2$沿着如图所示方向运动，绳中的张力及小球对斜面的压力各是多大？

（2）当斜面的加速度至少为多大时，小球将脱离斜面？



# 例3

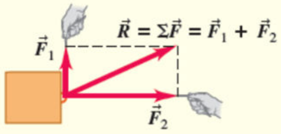
一个高楼擦窗工人利用滑轮-吊桶装置上升。
（1）要让自己匀速慢慢下降，他需要用多大的力拉绳；
（2）如果他放松些，使拉力减少10%，他的加速度将多大？设人和吊桶的总质量为75 kg。



## CHAPTER 4 SUMMARY

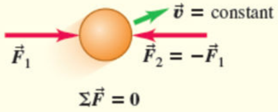
**Force as a vector:** Force is a quantitative measure of the interaction between two objects. It is a vector quantity. When several external forces act on an object, the effect on its motion is the same as if a single force, equal to the vector sum (resultant) of the forces, acts on the object. (See Example 4.1.)

$$\vec{R} = \sum\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \quad (4.1)$$




**The net external force on an object and Newton's first law:** Newton's first law states that when the vector sum of all external forces acting on a object (the *net external force*) is zero, the object is in equilibrium and has zero acceleration. If the object is initially at rest, it remains at rest; if it is initially in motion, it continues to move with constant velocity. This law is valid in inertial frames of reference only. (See Examples 4.2 and 4.3.)

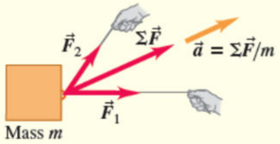
$$\sum\vec{F} = 0 \quad (4.3)$$




**Mass, acceleration, and Newton's second law:** The inertial properties of an object are characterized by its *mass.* Newton's second law states that the acceleration of an object under the action of a given set of external forces is directly proportional to the vector sum of the forces (the *net force*) and inversely proportional to the mass of the object. Like Newton's first law, this law is valid in inertial frames of reference only. In SI units, the unit of force is the newton (N), equal to 1 kg · m/s$^2$. (See Examples 4.4 and 4.5.)

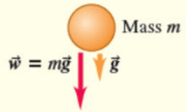
$$\sum\vec{F} = m\vec{a} \quad (4.6)$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \quad (4.7)$$



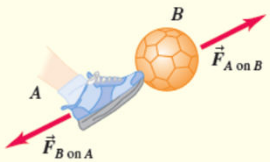

**Weight:** The weight $\vec{w}$ of an object is the gravitational force exerted on it by the earth. Weight is a vector quantity. The magnitude of the weight of an object at any specific location is equal to the product of its mass $m$ and the magnitude of the acceleration due to gravity $g$ at that location. The weight of an object depends on its location; its mass does not. (See Examples 4.6 and 4.7.)

$$w = mg \quad (4.8)$$

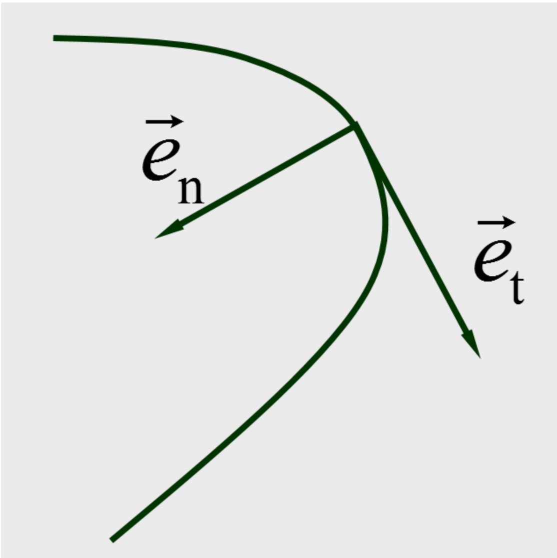



**Newton's third law and action–reaction pairs:** Newton's third law states that when two objects interact, they exert forces on each other that are equal in magnitude and opposite in direction. These forces are called action and reaction forces. Each of these two forces acts on only one of the two objects; they never act on the same object. (See Examples 4.8–4.11.)

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \quad (4.10)$$



# 牛顿第二定律的分量形式：自然坐标系




$$\vec{F} = F_{\mathrm{t}}\vec{e}_{\mathrm{t}} + F_{\mathrm{n}}\vec{e}_{\mathrm{n}}$$

$$\vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{e}_{\mathrm{t}} + \frac{v^2}{\rho}\vec{e}_{\mathrm{n}}$$

$$F_{\mathrm{t}} = ma_{\mathrm{t}} = m\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$F_{\mathrm{n}} = ma_{\mathrm{n}} = m\frac{v^2}{\rho}$$

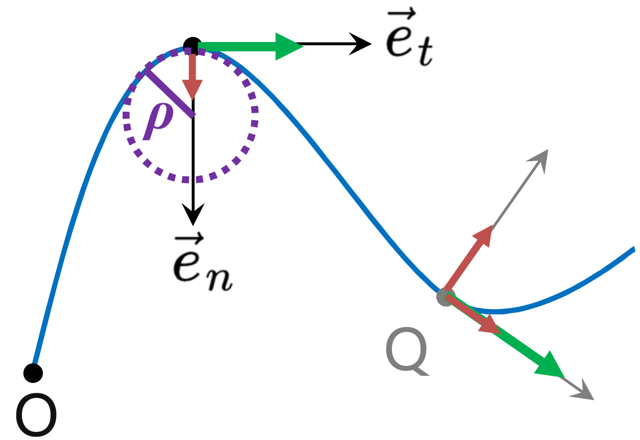
速度矢量的大小（速率）：$v=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\frac{ds}{dt}$

速度矢量的方向总是沿着为轨迹的切向，表示为：$\vec{v}=\frac{ds}{dt}\vec{e}_t$

加速度矢量：$\vec{a}=a_1\vec{e}_t+a_2\vec{e}_n$

$a_1$:切向加速度，改变速度的大小

$a_2$:法向加速度，改变速度的方向




$$a_1=\frac{dv}{dt}\qquad a_2=v\frac{d\theta}{dt}=v\frac{ds}{dt}\frac{d\theta}{ds}=\frac{v^2}{\rho}$$

其中，$\rho=\frac{ds}{d\theta}=\left|\frac{ds}{d\theta}\right|$　　为质点处轨迹的曲率半径，始终为正

# 惯性与惯性质量

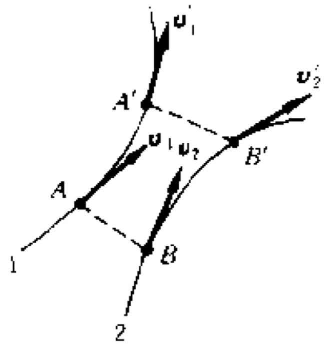
**惯性(inertia)**：物体保持运动状态的特性

**惯性质量(Inertial mass):**物体惯性大小的度量

惯性质量的操作定义：

（马赫，1867，《关于质量的定义》）

两个相互作用的物体的加速度的负比值）




实验上我们发现

（1）在任意给定的时间间隔内，这两个速度的变化 $\Delta \boldsymbol{v}_1$ 和 $\Delta \boldsymbol{v}_2$ 方向相反；

（2）在 $\Delta t \to 0$ 的极限下，$\Delta \boldsymbol{v}_1 \to d\boldsymbol{v}_1$ 和 $\Delta \boldsymbol{v}_2 \to d\boldsymbol{v}_2$ 排在两质点的瞬时联线上。

（3）不论时间间隔 $\Delta t$ 如何，速度的变化 $\Delta \boldsymbol{v}_1$ 和 $\Delta \boldsymbol{v}_2$ 的大小之比总是一样的。因此，我们可以写：

$$\Delta \boldsymbol{v}_1 = -K\Delta \boldsymbol{v}_2, \quad (2.1)$$

式中 $K$ 对于每一对质点是相同的，而与它们怎样运动无关。

（4）当我们分别用质点 2 和 3 与质点 1 进行实验时，(2.1) 式中的常数 $K$ 分别等于 $K_{12}$ 和 $K_{13}$，若用质点 3 和质点 2 直接做实验时，则有

$$K_{23} = K_{13}/K_{12}. \quad (2.2)$$

关于惯性质量定义的理想实验

惯性质量 和 引力质量 的差别?

**惯性质量，$m_{惯}$**

根据牛顿第二定律可知，对某一物体所用的力跟那个物体所得到的加速度的比值是一个恒量，这个恒量是表明那个物体性质的一个物理量，叫做那个物体的惯性质量，通常简称质量。

- 惯性大的物体质量大，惯性小的物体质量小。
- 惯性质量是物体惯性大小的量度。因惯性是物体固有的性质，故惯性质量也是物体固有的。
- 一般说来（不考虑相对论效应）惯性质量是不随外界任何其他条件而改变。
- 一个物体的惯性质量决定它受力时的加速度。

**引力质量，$m_{引}$**

在牛顿的万有引力定律中，也引入物体质量的概念，其中质量反映了物体间引力的大小，是物体引力性质的量度。因而，物理学史上曾把这种反映物体间万有引力大小的量称为引力质量，它与惯性质量是完全不同的概念。

尽管惯性质量与引力质量是从不同的定义出发，反映不同的物理性质，但数值上它们是相等的。实验结果表明，在$10^{-14}$的精度上，没有发现各种物质（$m_{惯}/m_{引}$）值的差别。这就从实验上向人们揭示：惯性质量精确地等于引力质量。

两种概念所定义的质量如此精确地相等，难道是偶然的吗？不会！爱因斯坦提出的广义相对论中，惯性质量就是引力质量。因此，我们可以不再区分这两种质量，而统称它们为质量。

# 牛顿第二定律的等效形式：动量定理

由于 $\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$

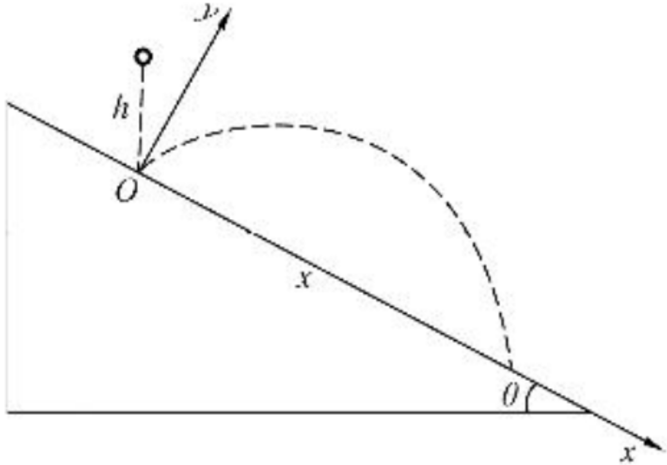
- 引入动量(momentum)$\vec{p} = m\vec{v}$,
- 因此有$\vec{F} = \frac{d\vec{p}}{dt}$，牛顿第二运动定律的另一表述是物体所受外力等于其动量对时间的变化率。

为什么要多此一举?

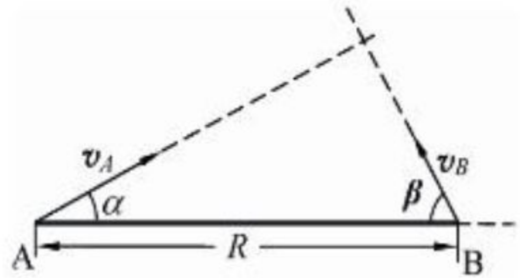
- 在引入相对论之后，物体的质量m将不再是一个恒定的值，而是随速度变化。$m = m_0/\sqrt{1-\frac{v^2}{c^2}}$，其中，$m_0$是物体静止时的质量。$F = m\frac{dv}{dt}$不再成立。然而$F = dp/dt$依然成立。
- 在引入量子力学之后，物体的速度和$F = dp/dt$将不具有确切的定义。但是动量仍然存在。

## HOMEWORK:（习题课讲解）

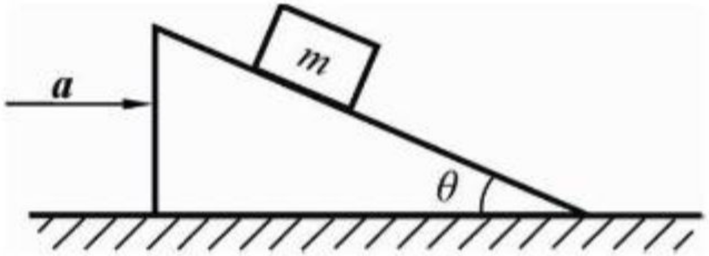
1-8．一弹性球直落在一斜面上，下落高度 $h = 20\text{cm}$，斜面对水平的倾角 $\theta = 30^\circ$，问它第二次碰到斜面的位置距原来的下落点多远(假设小球碰斜面前后速度数值相等，碰撞时人射角等于反射角)。




1-5. 如图所示，两船 $A$ 和 $B$ 相距 $R$，分别以速度 $\boldsymbol{v}_A$ 和 $\boldsymbol{v}_B$ 匀速直线行驶，它们会不会相碰?若不相碰，求两船相靠最近的距离．图中 $\alpha$ 和 $\beta$ 为已知。



2-5．如图所示一倾角为$\theta$的斜面放在水平面上，斜面上放一木块，两者间摩擦系数为$\mu(<\tan\theta)$。为使木块相对斜面静止，求斜面加速度$a$的范围。




2-8．在光滑的水平面上设置一竖直的圆筒，半径为$R$，一小球紧靠圆筒内壁运动，摩擦系数为$\mu$，在$t=0$时，球的速率为$v_0$，求任一时刻球的速率和运动路程。