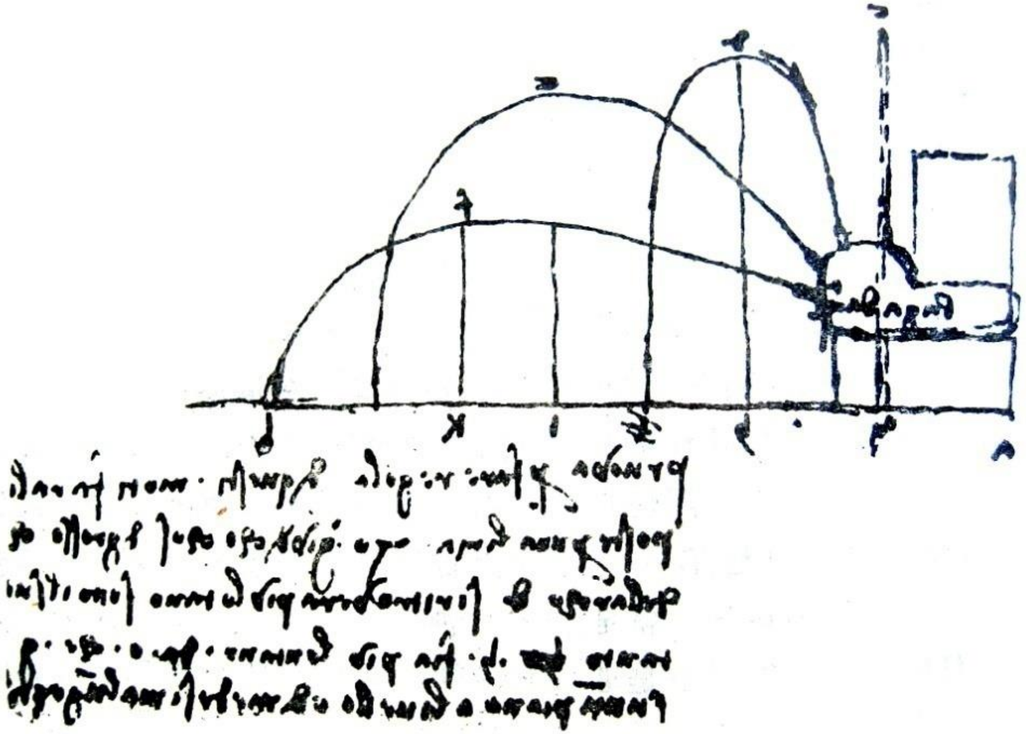
普通物理I PHYS1181.03 第2章

质点运动学
kinematics

Kinematics: **the part of mechanics that describe motion.**
运动学：描述物体运动

Dynamics: why objects move in different ways
动力学：运动状态维持或改变的原因



达·芬奇手稿中关于弹道的研究

力学的研究对象

物理与力学

Physics 物理 ⟶ “物” “理” ⟶ 万事万物变化规律

古希腊人把所有对自然界的观察和思考，笼统地包含在一门学问里，即自然哲学。

物理学是最直接地关心自然界最基本规律的，是探讨物质的结构和运动基本规律的学科。

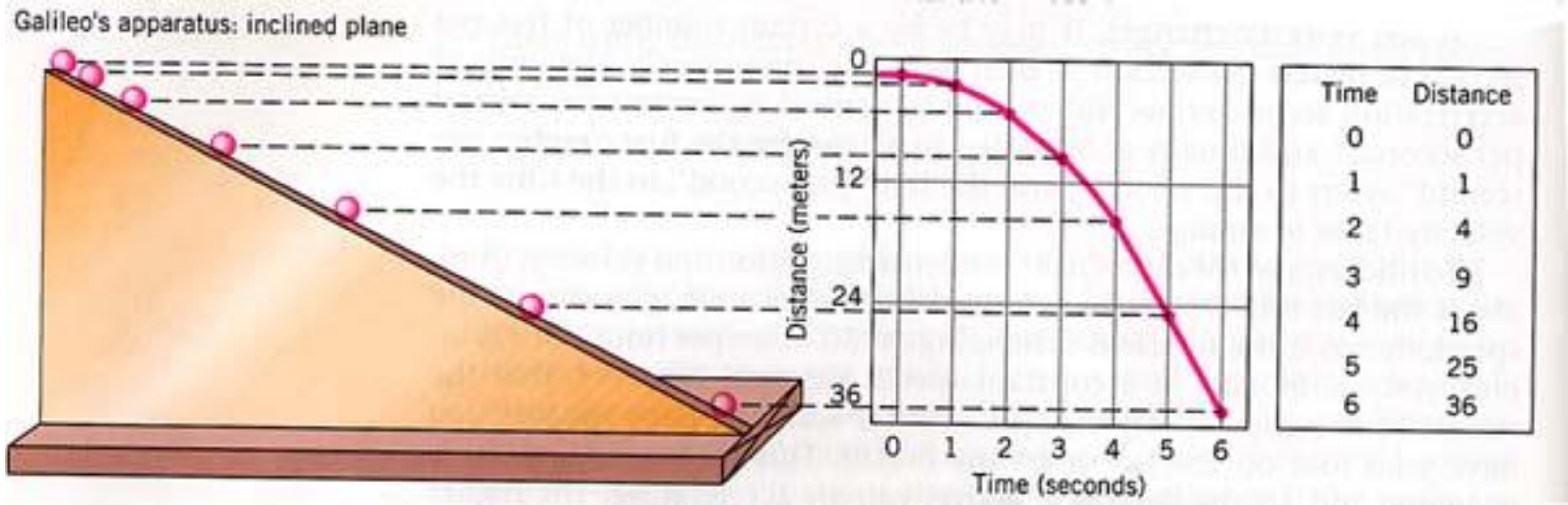
相互作用 ⟶ 力 ⟶ “力学”

力学是基础课的基础，以经典内容为主的，是学习物理学的重要基础。

时间、空间和牛顿力学的绝对量

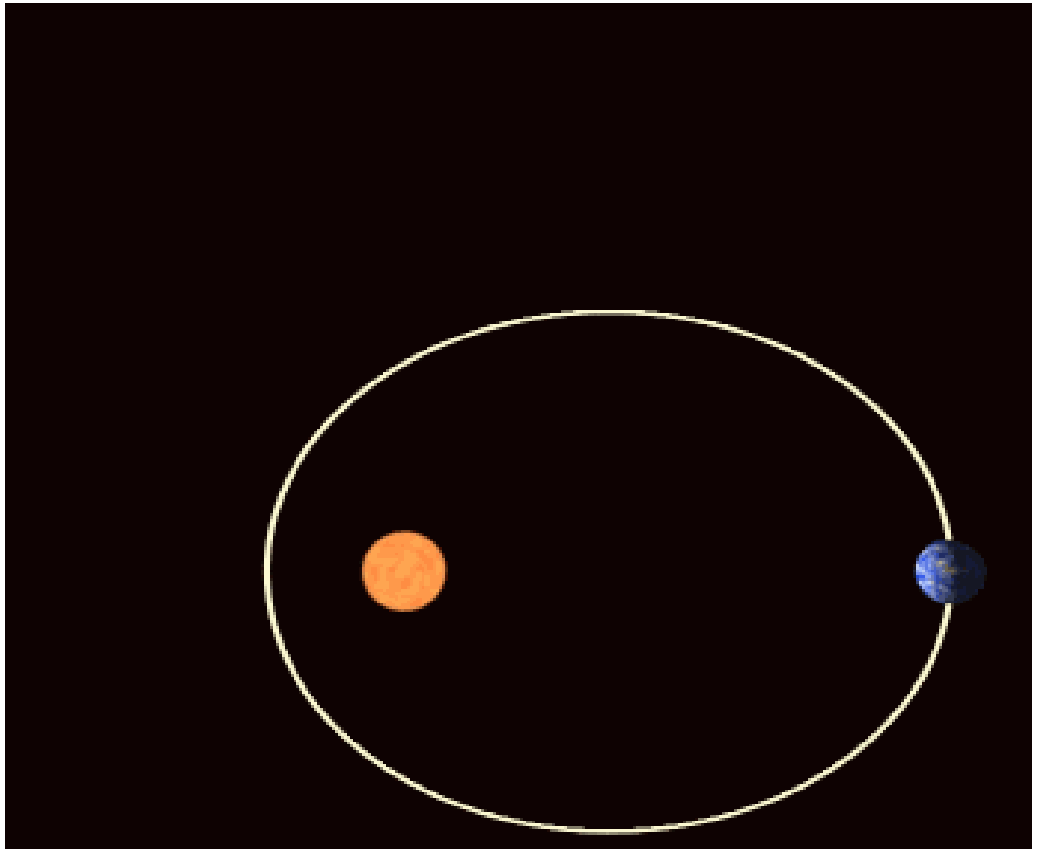
描述物体的运动，要用时间和空间这两个概念。

机械运动的定义：物体的空间位置随时间的变化。



什么是时间？什么是空间？

运动



Apsidal precession：

From Newton to General relativity

整个轨道随着时间也在“运动”

（1）基本的概念

Displacement: 位移
Velocity: 速度
Acceleration: 加速度

} 矢量

Time: 时间 } 标量

速度：位移相对于时间的改变
加速度：速度相对于时间的改变

一维情况：

- 平均 → 瞬时极限
- 位移、速度 v.s. 时间图
- 微分、积分

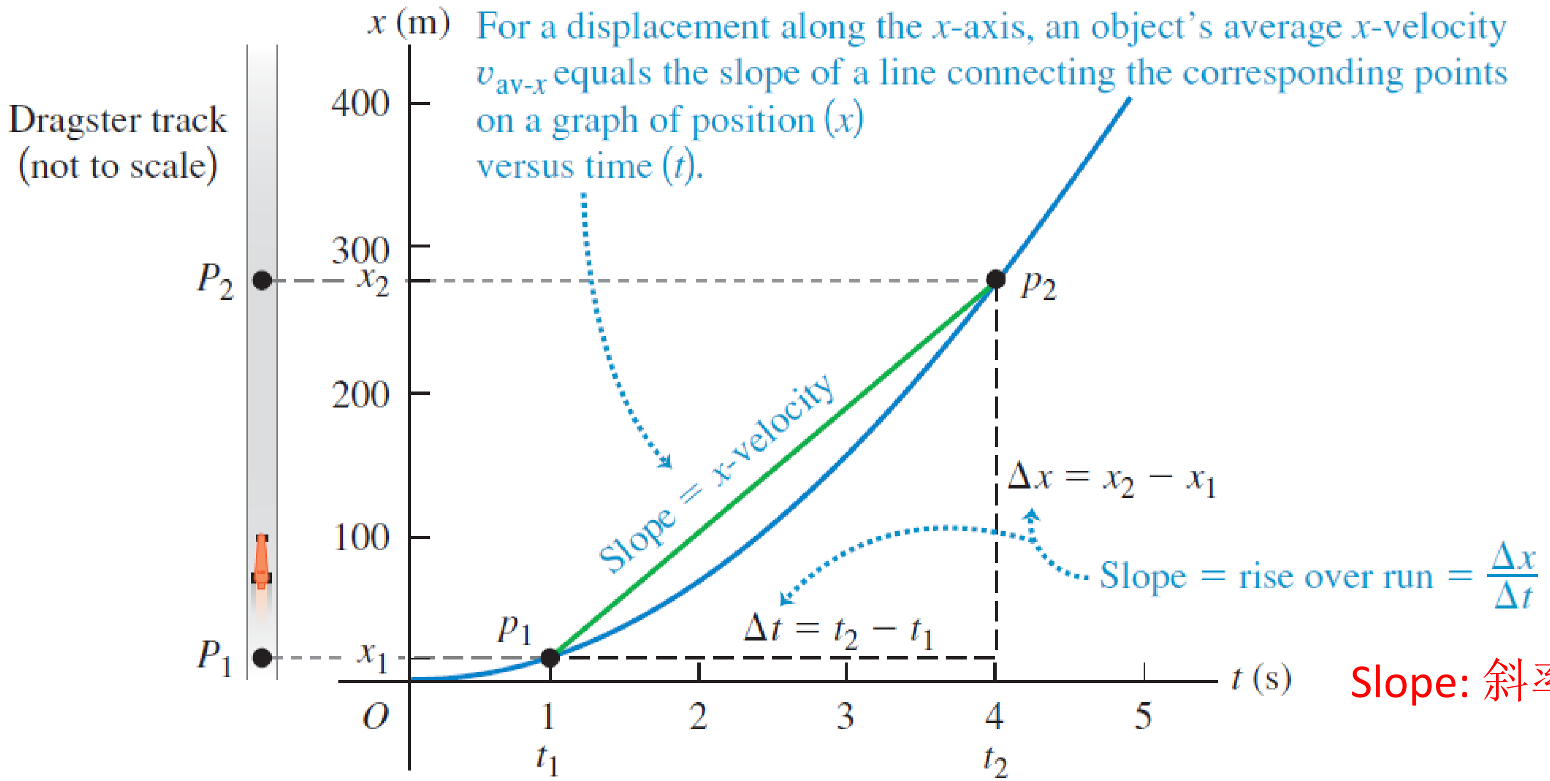
一维的情形

平均速度

从起点到终点，经历的位移

平均速度 (average velocity)

$$v_{av\text{-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

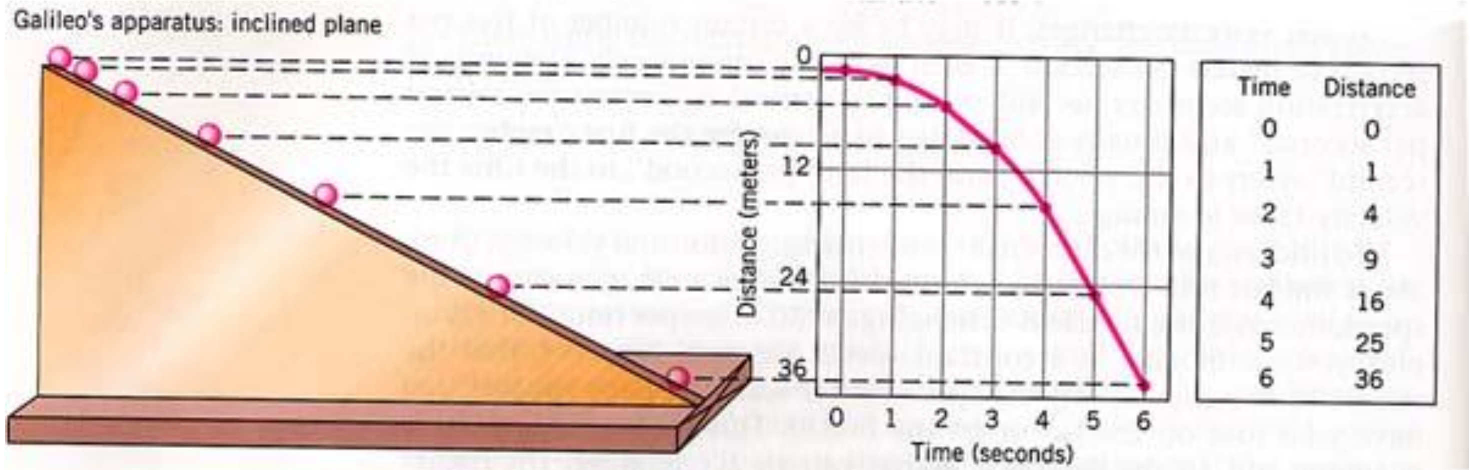


Slope: 斜率

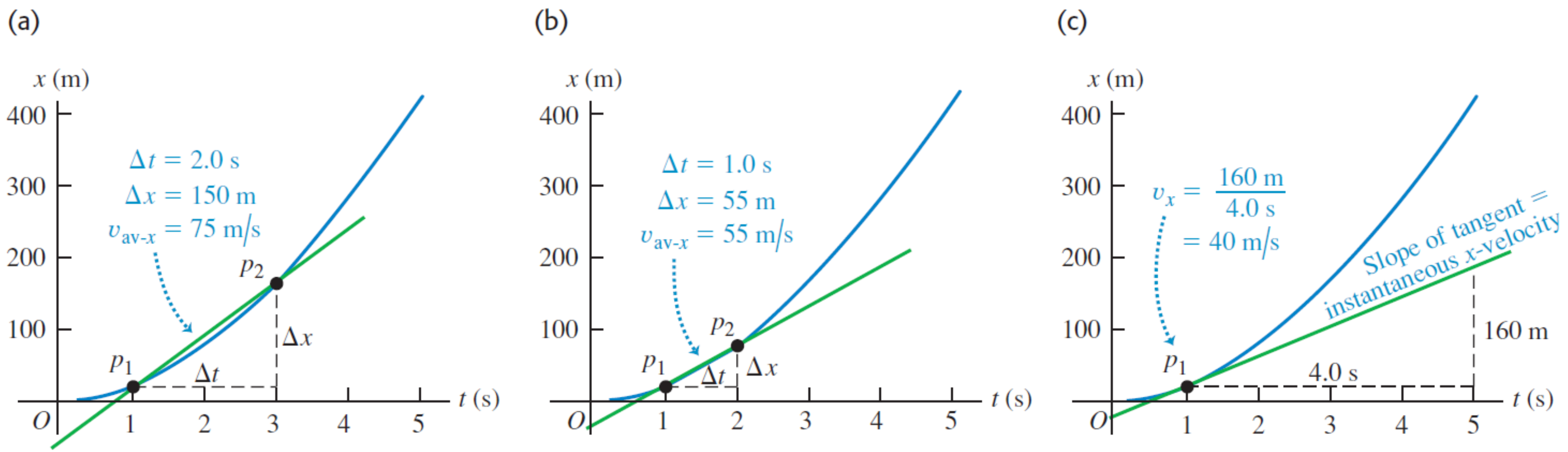
列举生活中直线运动的例子

直线匀速运动

直线变速运动



在位移-时间图上描述瞬时速度



As the average x-velocity $v_{av\text{-}x}$ is calculated over shorter and shorter time intervals ...

... its value $v_{av\text{-}x} = \Delta x/\Delta t$ approaches the instantaneous x-velocity.

The instantaneous x-velocity v_x at any given point equals the slope of the tangent to the x-t curve at that point.

瞬时速度

为了能够刻画一个物体在任意时刻的运动快慢，我们需要将用于测量平均速度的时间不停缩短，再缩短，直到趋向于一个无穷小的时间间隔，这个时候我们就能够得到在一个特定时刻物体的平均速度，称为瞬时速度(instantaneous velocity)。

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \text{(instantaneous } x\text{-velocity, straight-line motion)}$$

数学记号，称作求x对于t的导数。dx和dt称做x和t这两个变量的微分
这种无穷小，求极限的观念和方法由牛顿和莱布尼兹分别引入，开创了微积分这个数学方法。

瞬时速度的改变：加速度

定义了瞬时速度后，速度就和位移一样，成为了时间的一个函数v(t)。
通过类似的方法，定义平均加速度和瞬时加速度。

平均加速度

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \quad \text{(average } x\text{-acceleration, straight-line motion)}$$

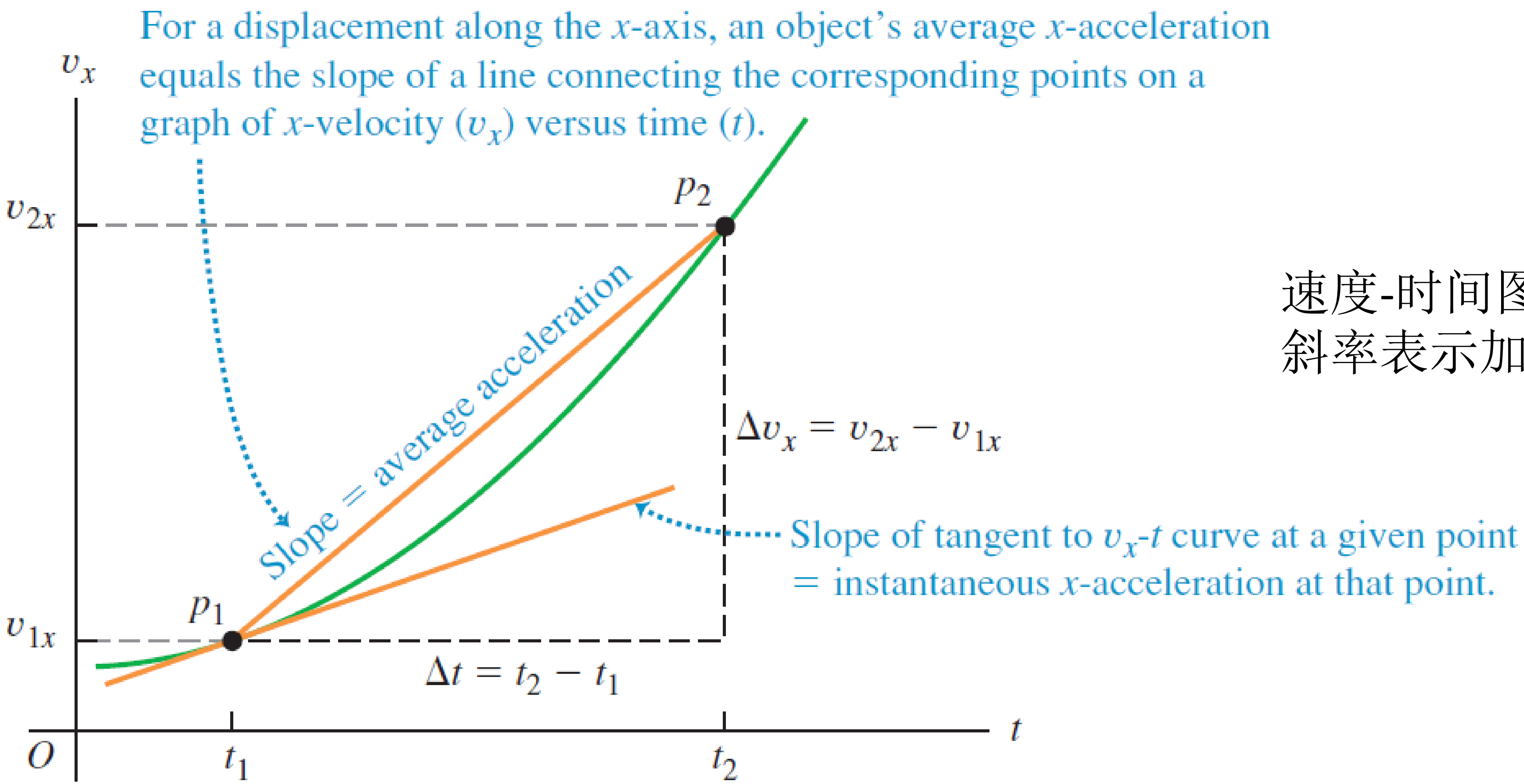
瞬时加速度

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad \text{(instantaneous } x\text{-acceleration, straight-line motion)}$$

加速度是位移关于时间的二次导数。记作

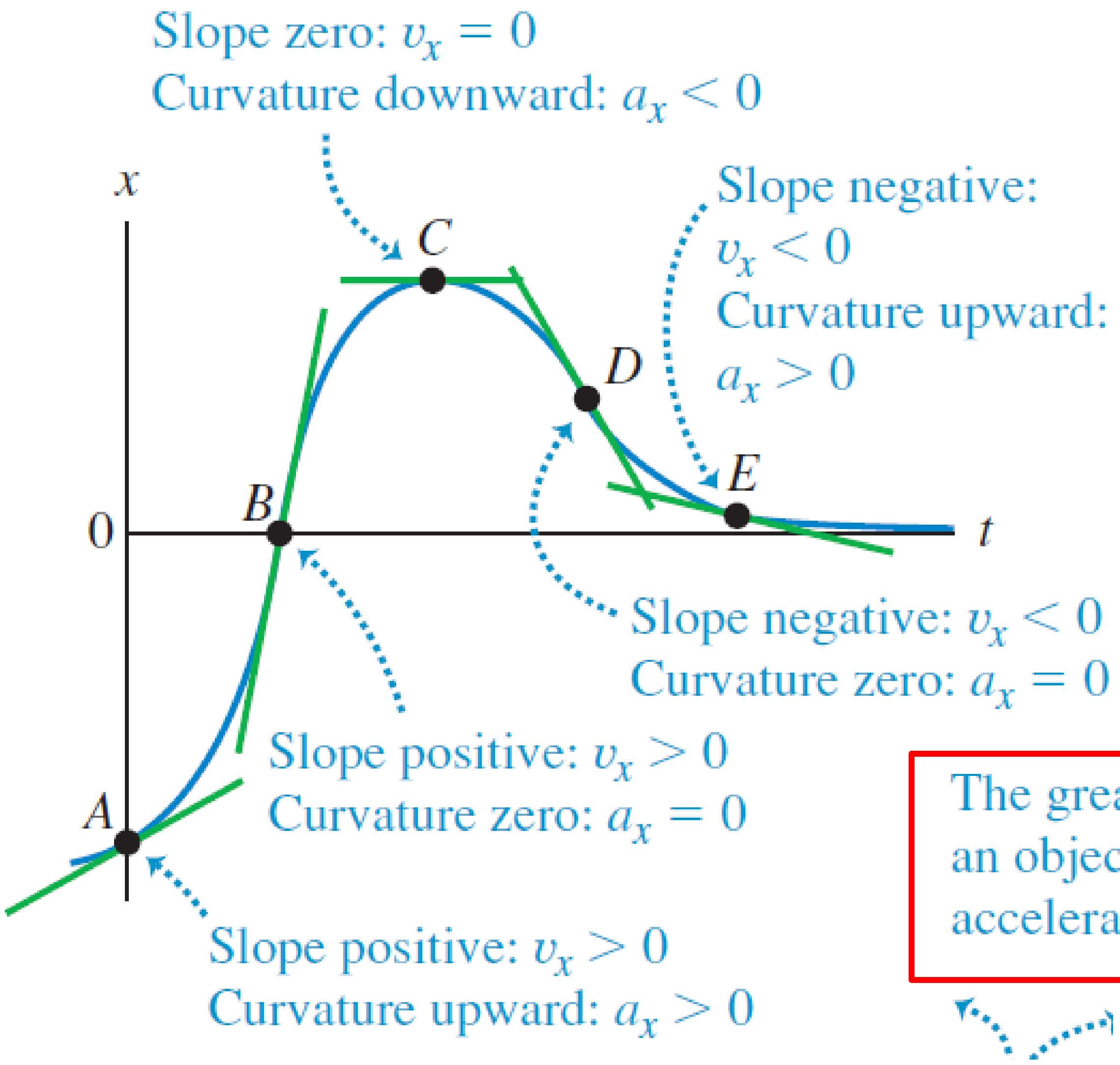
$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

在速度-时间图上看出加速度



速度-时间图
斜率表示加速度

在位移-时间图上看出加速度



The greater the curvature (upward or downward) of an object's x-t graph, the greater is the object's acceleration in the positive or negative x-direction.

匀加速直线运动 Motion with constant acceleration

***x*-velocity** at time t of a particle with **constant *x*-acceleration**

x-velocity of the particle at time 0

$$v_x = v_{0x} + a_x t \qquad (2.8)$$

Constant *x*-acceleration of the particle

Time

$$v_{av\text{-}x} = \tfrac{1}{2}(v_{0x} + v_x) \quad \text{(constant } x\text{-acceleration only)}$$

Position of the particle at time 0

Time

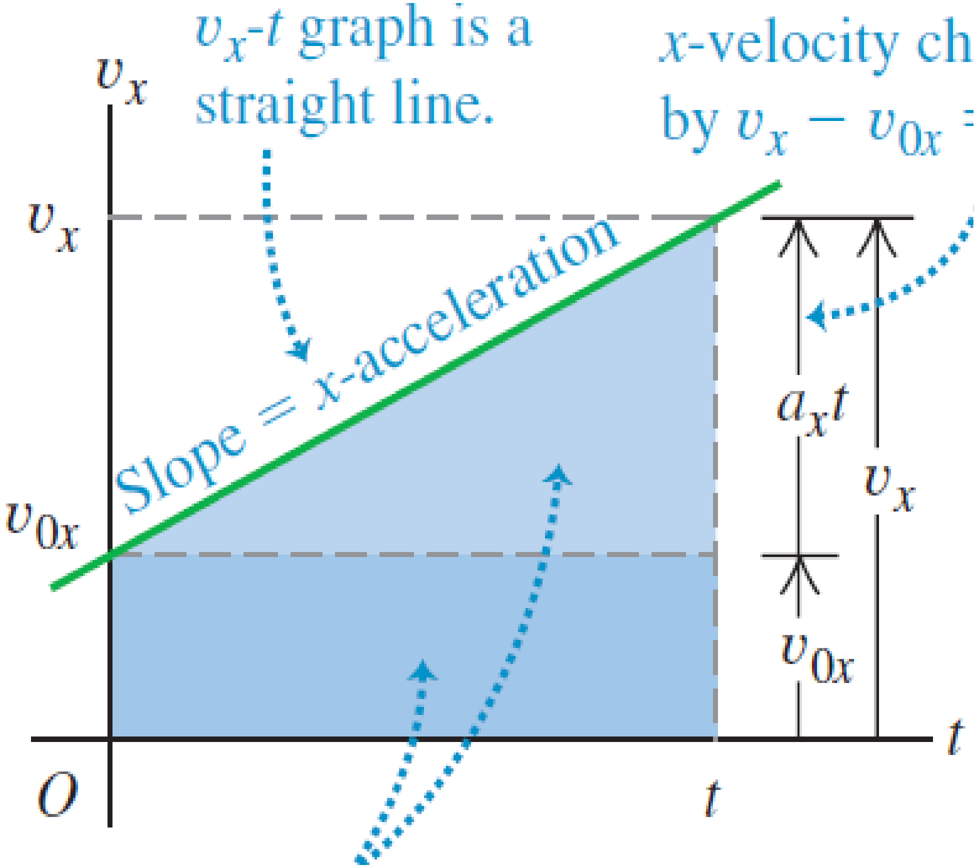
Position at time t of a particle with **constant *x*-acceleration**

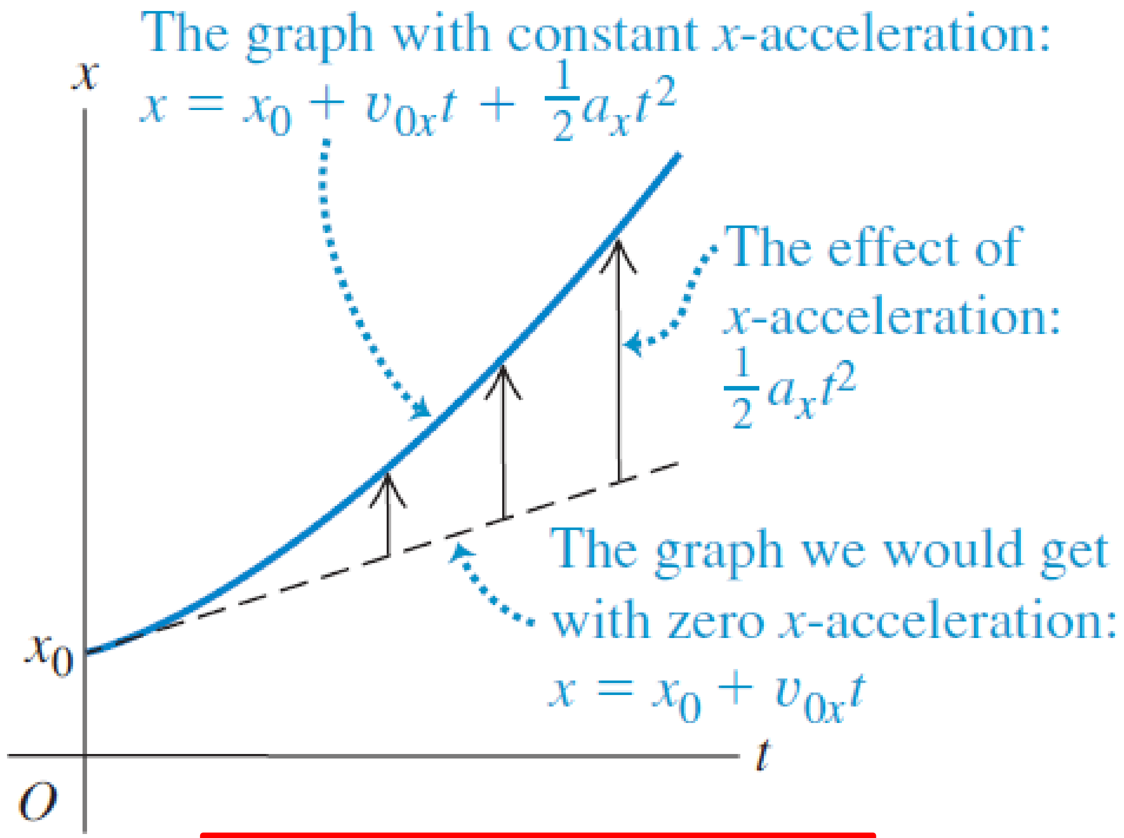
$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad (2.12)$$

x-velocity of the particle at time 0

Constant *x*-acceleration of the particle

匀加速直线运动 Motion with constant acceleration

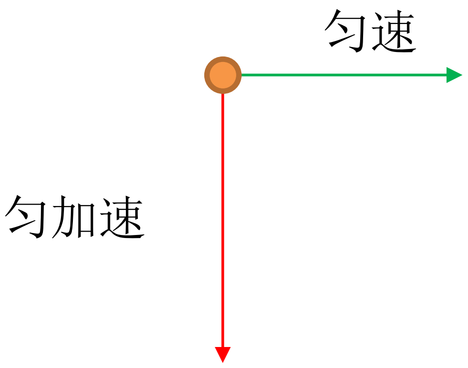




$$v_x = v_{0x} + a_x t$$

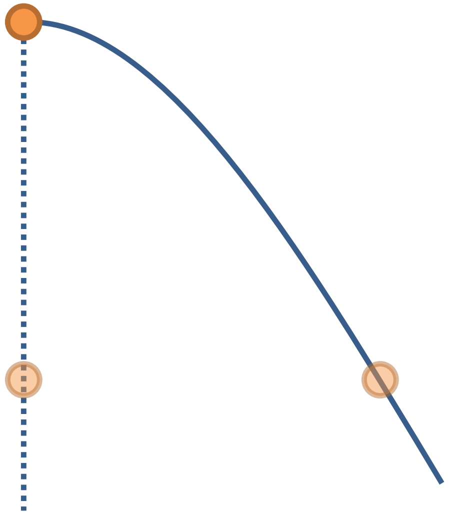
$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

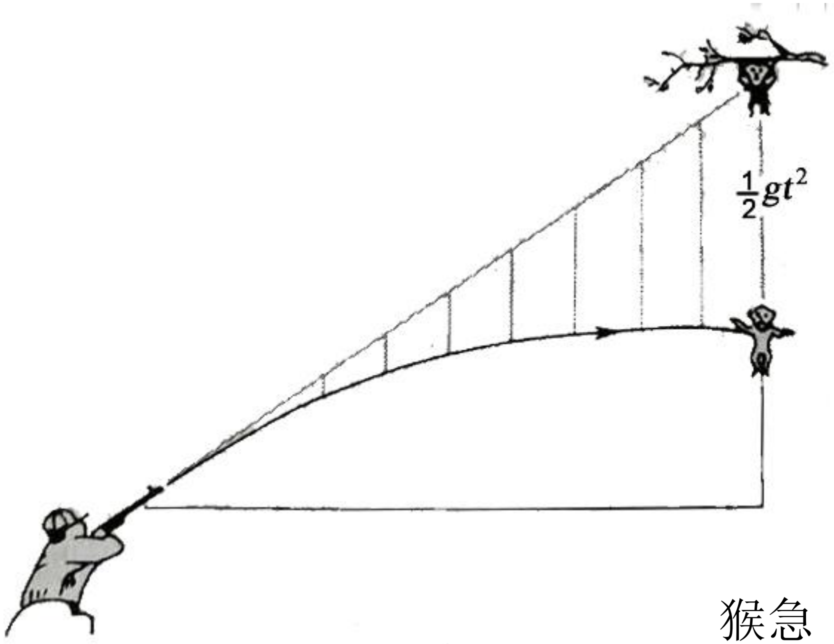
抛物线



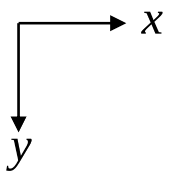
重力加速度 g

$$y(t) = \frac{1}{2}gt^2$$





猴急



小結:

极限 → 微分

瞬时速度:

The **instantaneous** ***x*-velocity** of a particle in **straight-line motion** ...

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

... equals the limit of the particle’s average *x*-velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle’s *x*-coordinate.

瞬时加速度:

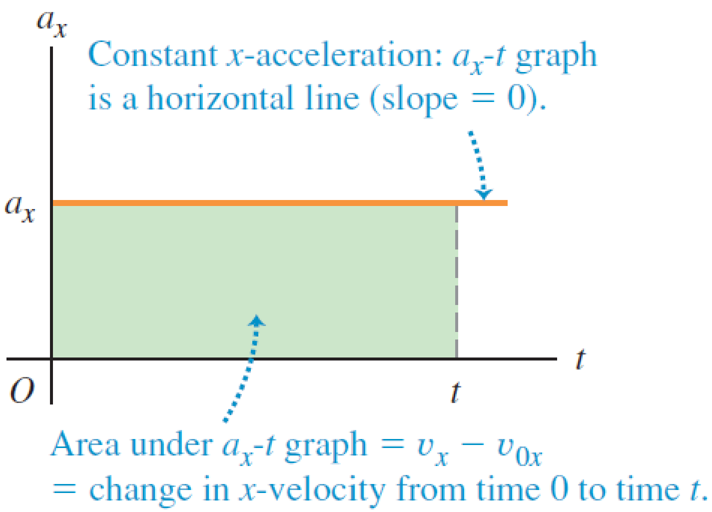
The **instantaneous** ***x*-acceleration** of a particle in **straight-line motion** ...

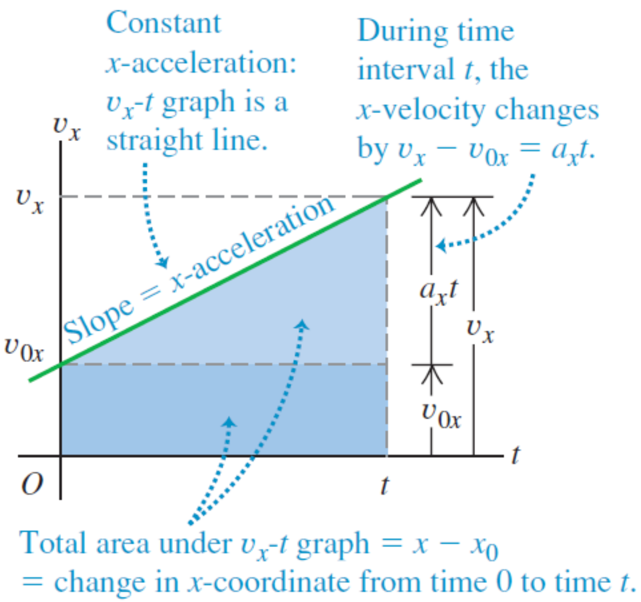
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

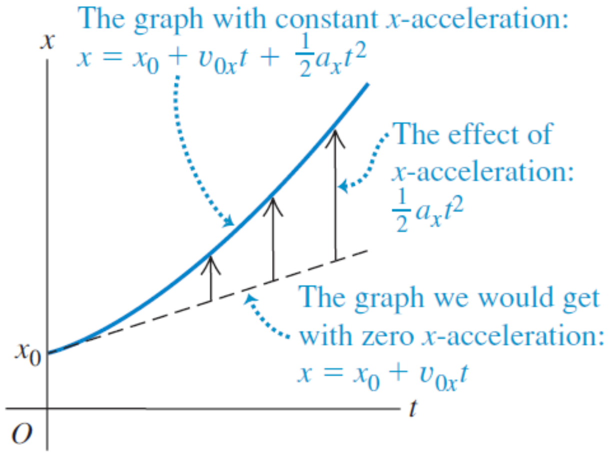
... equals the limit of the particle’s average *x*-acceleration as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle’s *x*-velocity.

例：匀加速直线运动







$a_x = a_0$ 为常数

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

加速度-时间 曲线　　**速度-时间** 曲线　　**位移-时间** 曲线

（2）微积分 ——现代科学和工程的基础

微积分--工科

高等数学– 理科

数学分析 – 数学

基本函数的微分

Derivatives: （导数）

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\ln ax = \frac{1}{x}$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax$$

$$\frac{d}{dx}(uv) = u\left(\frac{dv}{dx}\right) + v\left(\frac{du}{dx}\right)$$ （微分乘法律）

$$\frac{d}{dx}(ax^m \pm bx^n) = amx^{m-1} \pm bnx^{n-1}$$

连锁律

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$ （微分连锁律） 链式法则

[例题]一质点沿*x*轴正向运动，假设该质点通过坐标为*x*时的速度大小为***kx***（***k***为正常量）。求（1）此时该质点的加速度；（2）该质点从$x = x_1$点出发运动到$x = x_2$点处所经历的时间间隔Δt。

[解]：（1）

$$\because \upsilon = kx$$

$$\therefore a = \frac{d\upsilon}{dt} = k\frac{dx}{dt} = k\upsilon = k^2 x$$

（2）

$$\because \frac{dx}{dt} = kx \Rightarrow \frac{dx}{x} = kdt$$

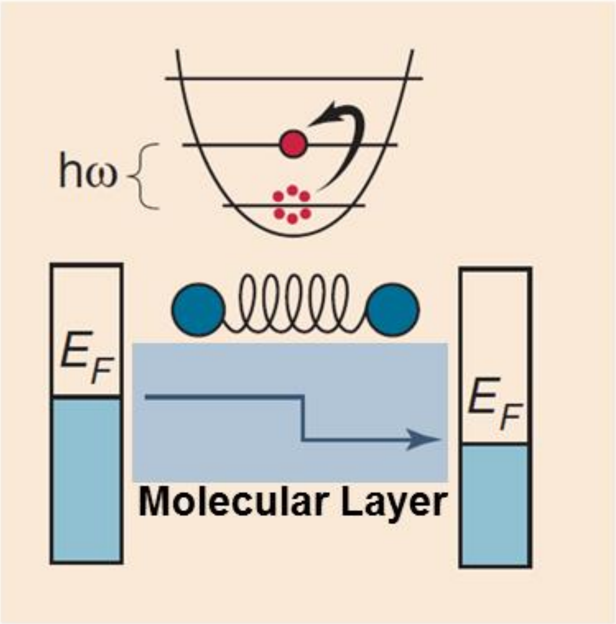
$$\therefore \Delta t = \frac{1}{k}\ln\frac{x_2}{x_1}$$

二阶导数

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

这是二阶微分的数学表达方式，和+， -， *, / 一样是一个运算符号！

不是“平方”操作。

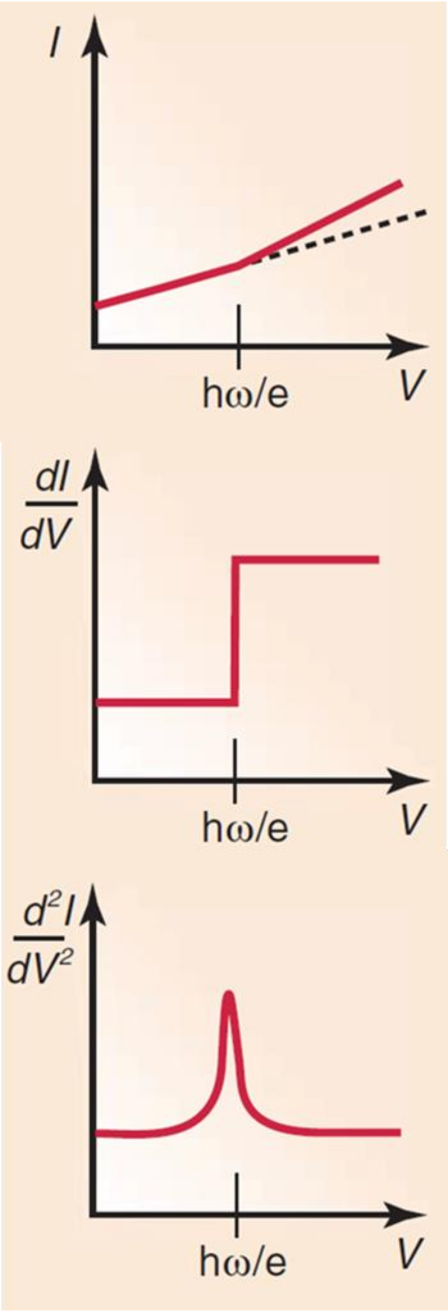


Electron transport through molecular layer by tunneling effect

Inelastic tunneling channel open when

$$eV = \hbar\omega_{vib}$$

V: bias of the junction
ω: frequency of the vibrational mode



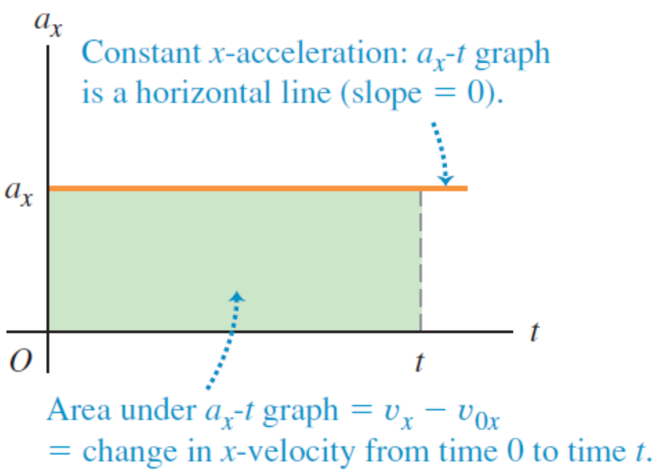
电流

微分电导

二阶微分电导

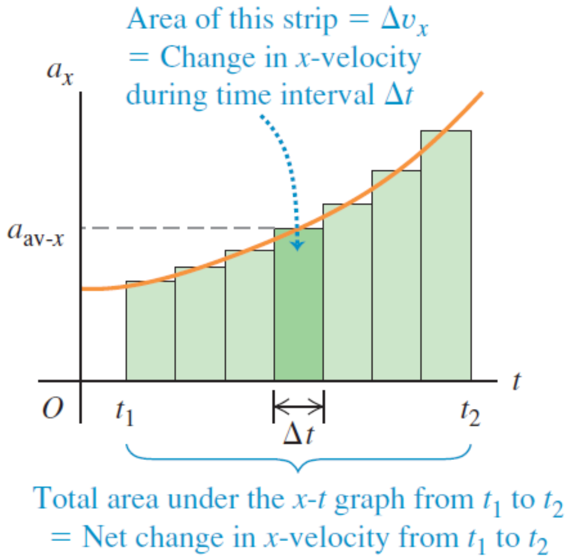
Current change by inelastic process is presented by its second derivative

积分：微分的逆运算



匀加速直线运动

$$v_x = v_{0x} + a_x t \qquad \text{(constant } x\text{-acceleration only)}$$



变加速直线运动

$$v_x - v_{0x} \approx \sum_i a_x(t_i)\Delta t = \lim_{\Delta t\to 0}\sum_i a_x(t_i)\Delta t$$

记作： $$v_x - v_{0x} = \int_{t_1}^{t_2} a_x(t)\, dt$$

基本函数的积分公式

Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$\int \frac{dx}{x} = \ln x$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

[例题]一质点沿x轴运动，其加速度随位置的变化关系为：$a=3x^2+\frac{1}{3}$，若x=0处，速度 $\upsilon_0=5m/s$，问：x=3m处的速率是多少？

[解]：

$$v_x - v_{0x} = \int_{t_1}^{t_2} a_x(t)\,dt$$

$$\because a = a(x) = a(x(t))$$

复合函数

$$\therefore a = \frac{d\upsilon}{dt} = \frac{d\upsilon}{dx}\frac{dx}{dt}$$

链式法则

$$= \upsilon\frac{d\upsilon}{dx} = 3x^2 + \frac{1}{3}$$

乘上dx

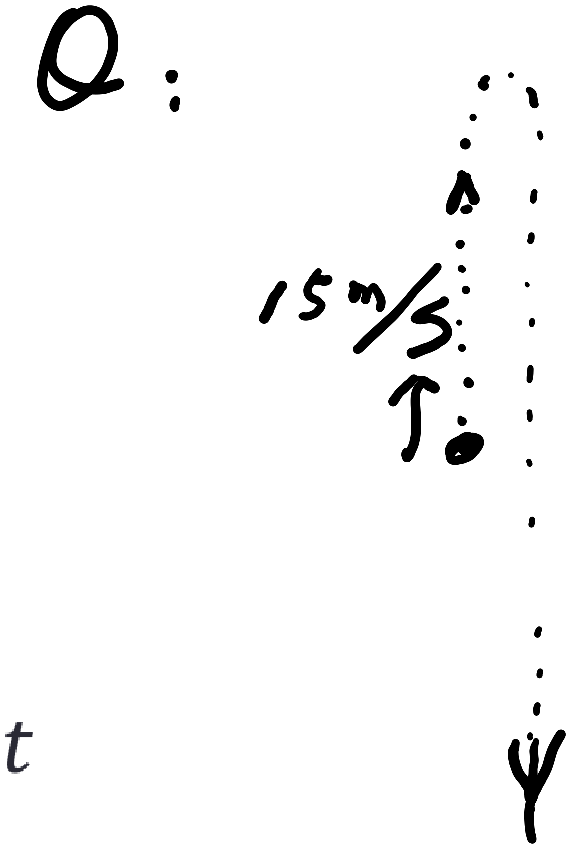
$$\Rightarrow \int_{\upsilon_0}^{\upsilon} \upsilon d\upsilon = \int_0^x \left(3x^2+\frac{1}{3}\right)dx$$

$$\Rightarrow \frac{1}{2}\left(\upsilon^2-\upsilon_0^2\right) = x^3 + \frac{1}{3}x$$

$$\therefore \upsilon\big|_{x=3m} = 9m/s$$

以自由落体为例，计算速度和距离

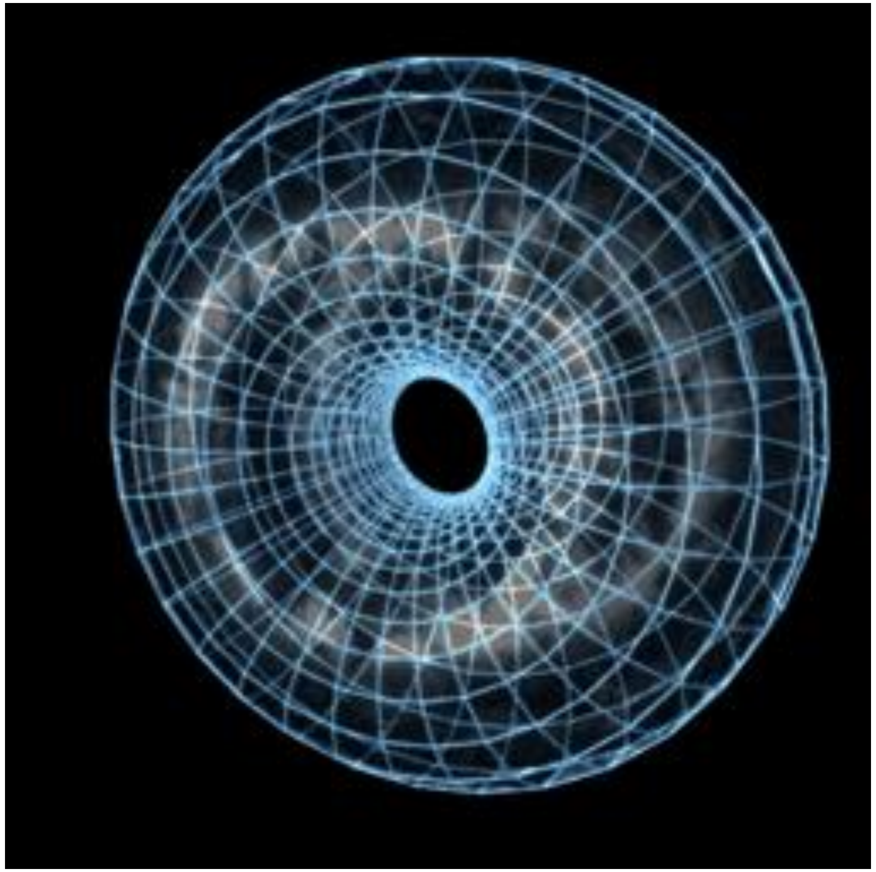
- 重力加速度g
- $v(t) = \int_0^t g dt$
- $s(t) = \int_0^t v(t) dt = \int_0^t (\int_0^t g dt) dt$



（3）位置矢量、速度矢量、加速度矢量

三维运动

人类可感知的世界是3维空间 + 1维时间

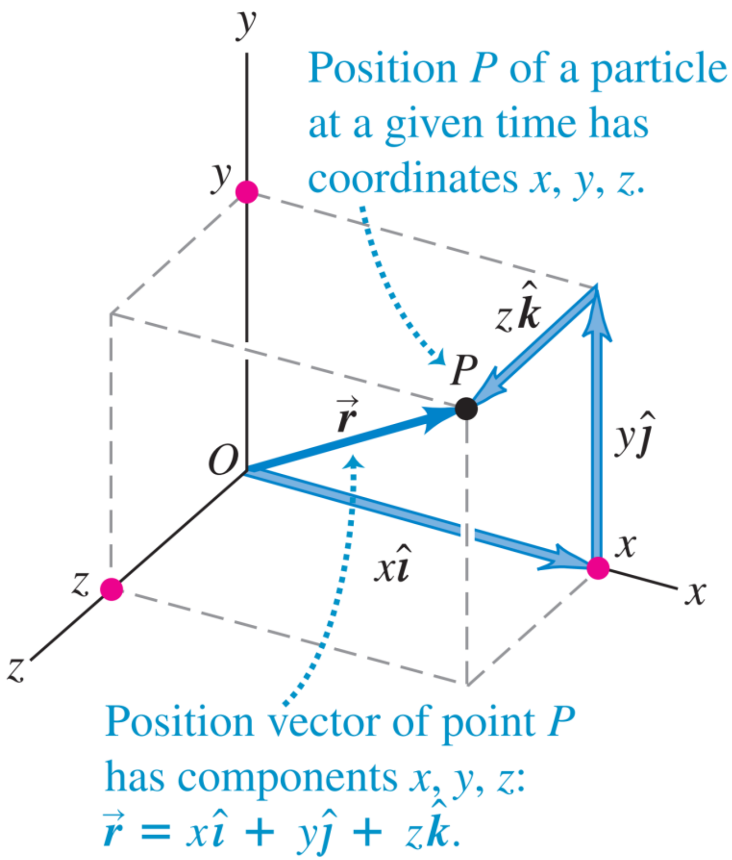


直角坐标系： x, y, z 三个正交坐标轴

Cartesian coordinates

引入新的概念：位矢

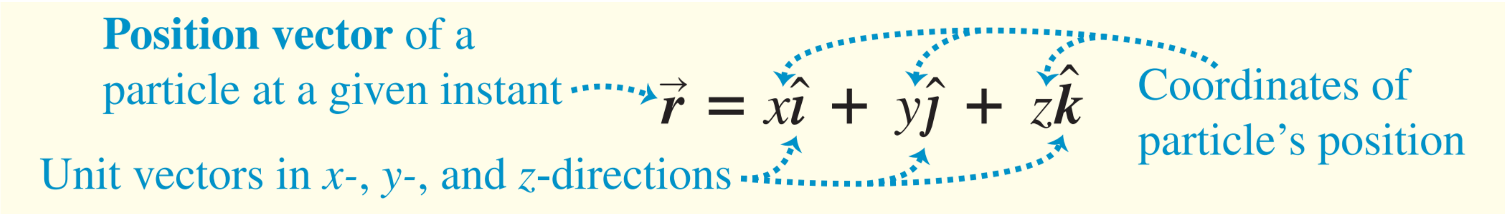
Position vector（位置矢量）



矢量的正确写法：

顶带箭头 或右下带坐标轴

位矢可以分解为x, y, z三个方向的分矢量：



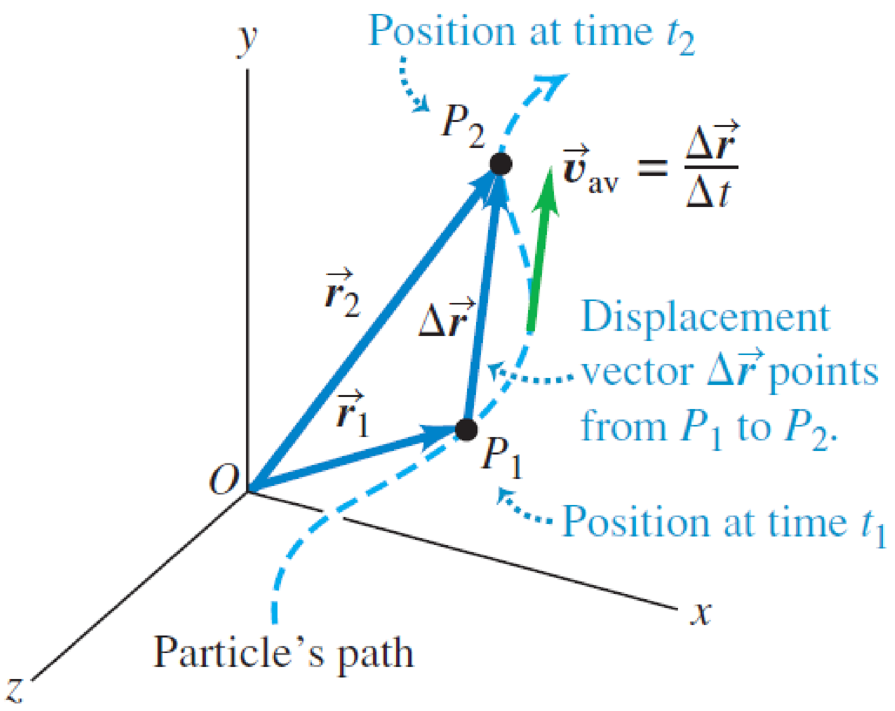
质点在时间 *t* 时位于 *P* 点：

位置矢量： $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

速度 矢量： $\vec{\upsilon}(t) = \frac{dx(t)}{dt}\vec{i} + \frac{dy(t)}{dt}\vec{j} + \frac{dz(t)}{dt}\vec{k}$

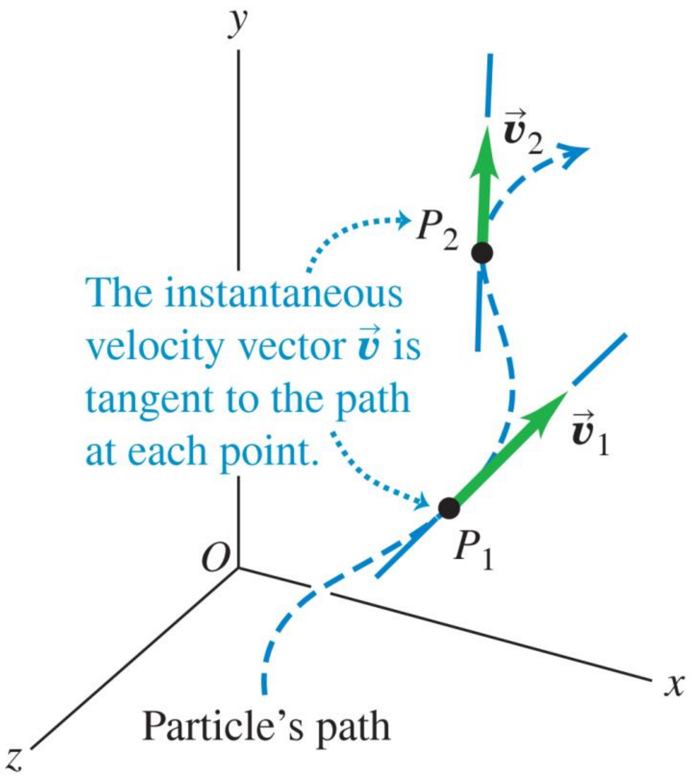
加速度 矢量： $\vec{a}(t) = \frac{d^2x(t)}{dt^2}\vec{i} + \frac{d^2y(t)}{dt^2}\vec{j} + \frac{d^2z(t)}{dt^2}\vec{k}$

平均速度矢量



$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

（瞬时）速度矢量



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

运动迭加原理（独立性原理）

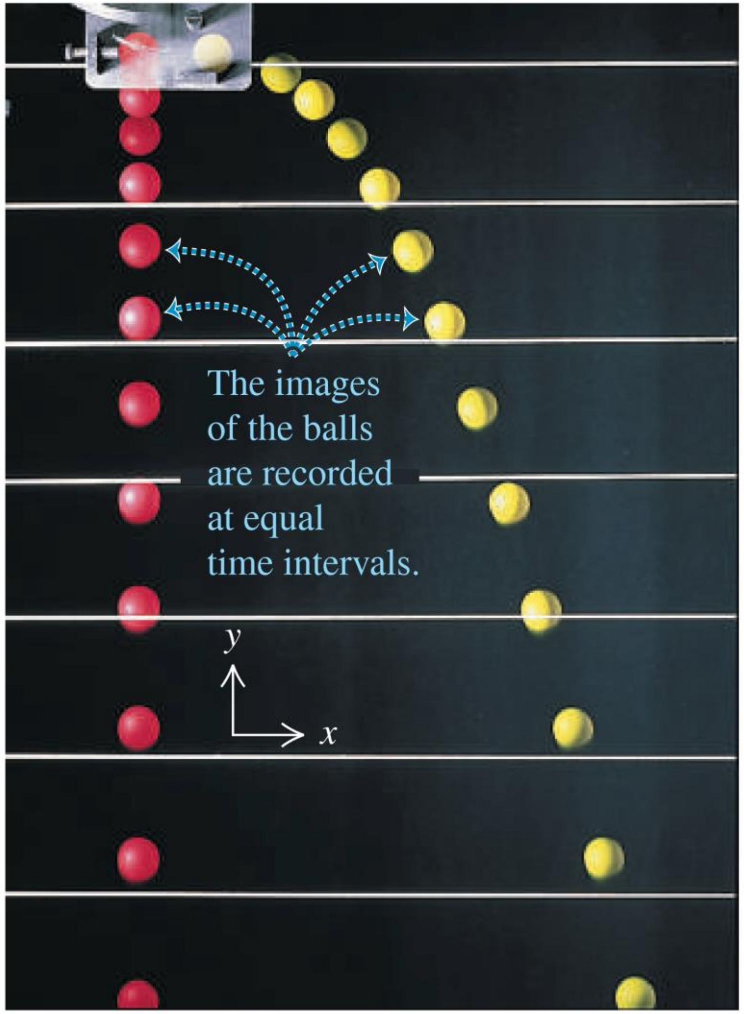
一个运动可看成几个独立进行的运动的迭加。

独立运动 —合成→ 合运动
独立运动 ←分解— 合运动

简单运动 ← 复杂运动

运动学的两类问题：

运动方程 —微分→ $\vec{v}$或$\vec{a}$
运动方程 ←积分— $\vec{v}$或$\vec{a}$（初始条件）



- At any time the two balls have different x-coordinates and x-velocities but the same y-coordinate, y-velocity, and y-acceleration.
- The horizontal motion of the yellow ball has no effect on its vertical motion.

速度可以分解为三个方向的分速度：

Each **component of** a particle's **instantaneous velocity vector** ...

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

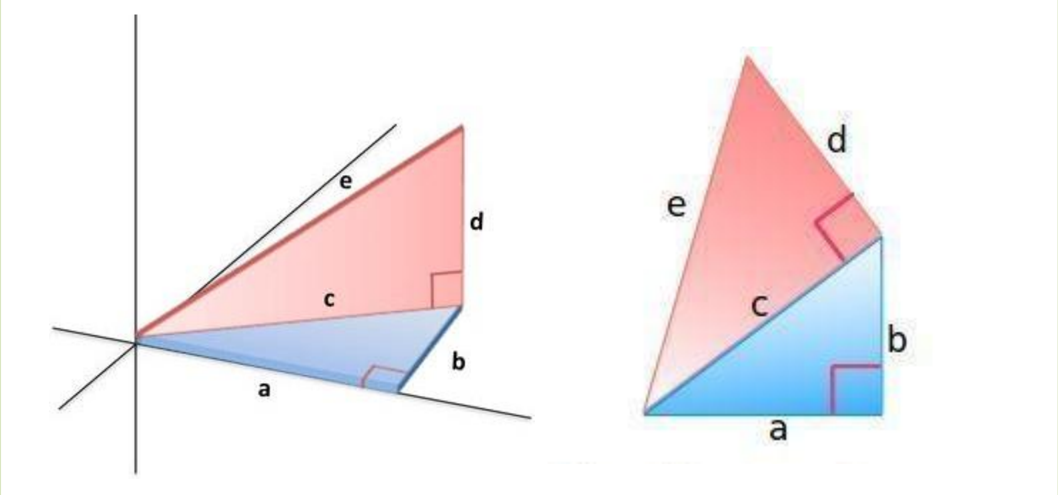
... equals the instantaneous rate of change of its corresponding coordinate.

速度矢量： $$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{\boldsymbol{i}} + v_y\hat{\boldsymbol{j}} + v_z\hat{\boldsymbol{k}}$$

根据 Pythagorean relationship（勾股定理）：

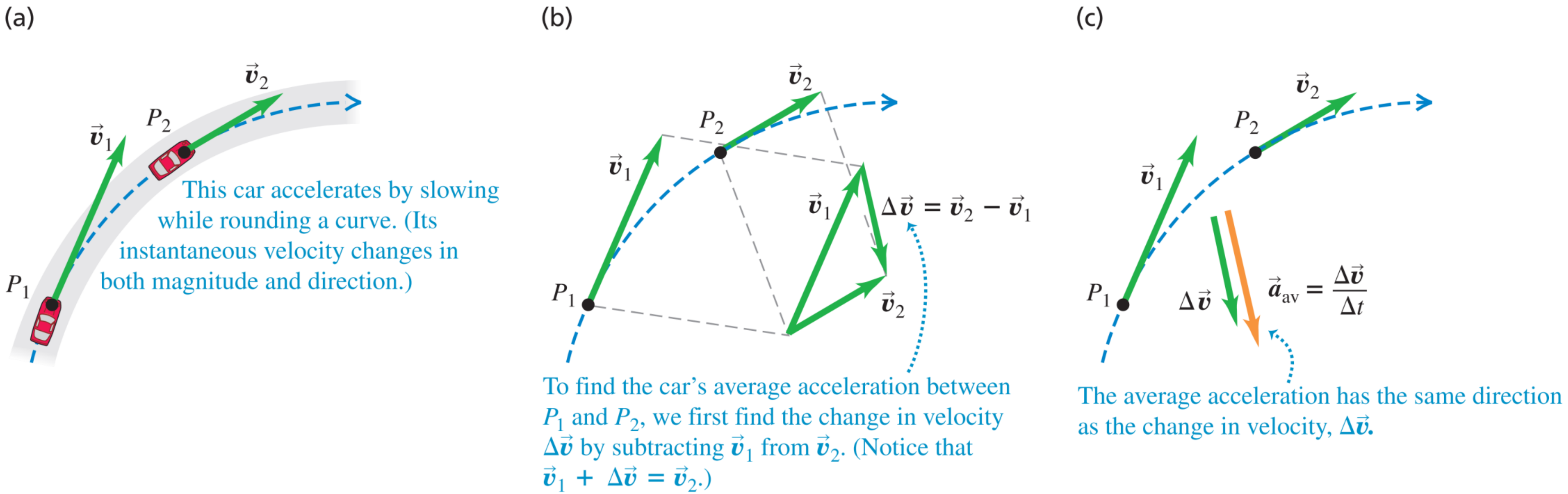
速度的大小：

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$



$$a^2 + b^2 = c^2$$
$$c^2 + d^2 = e^2$$
$$\Rightarrow a^2 + b^2 + d^2 = e^2$$

平均加速度矢量：



Change in the particle's velocity

Average acceleration vector of a particle during time interval from t_1 to t_2

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

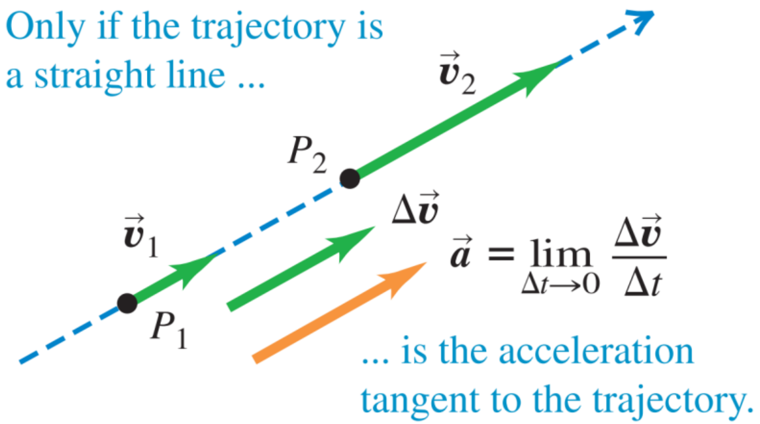
Final velocity minus initial velocity

Time interval

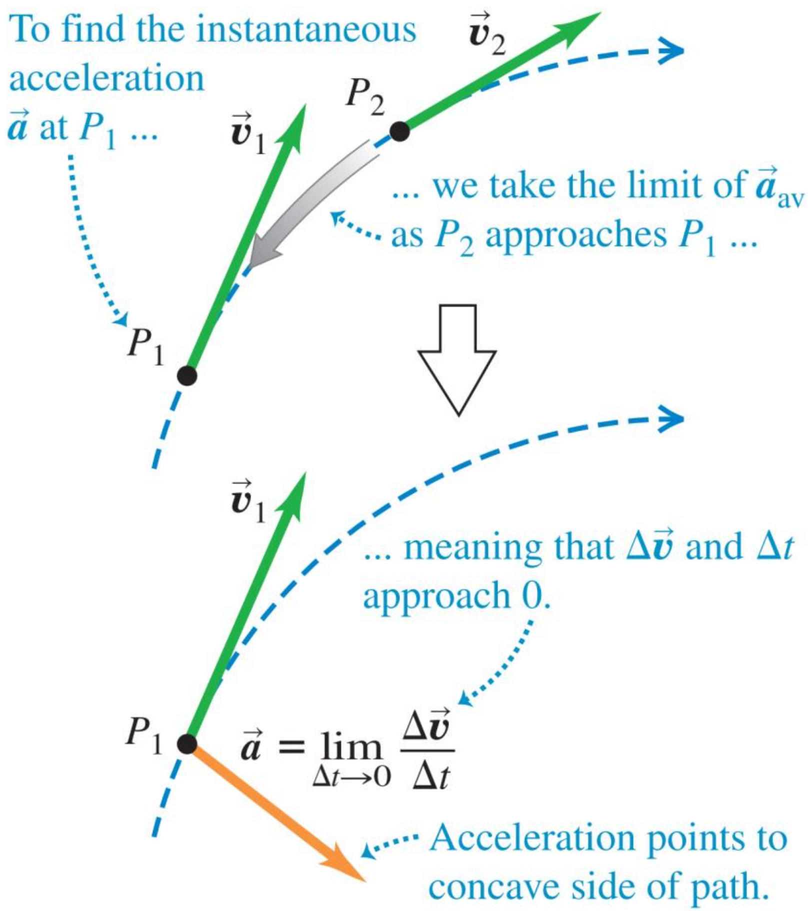
Final time minus initial time

（瞬时）加速度矢量：

(a) Acceleration: straight-line trajectory



(b) Acceleration: curved trajectory



The **instantaneous acceleration vector** of a particle ...

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

... equals the limit of its average acceleration vector as the time interval approaches zero ...

... and equals the instantaneous rate of change of its velocity vector.

和位置矢量、速度矢量类似，在笛卡尔坐标系中，

加速度矢量也可以分解为 x, y, z 三个方向的分量：

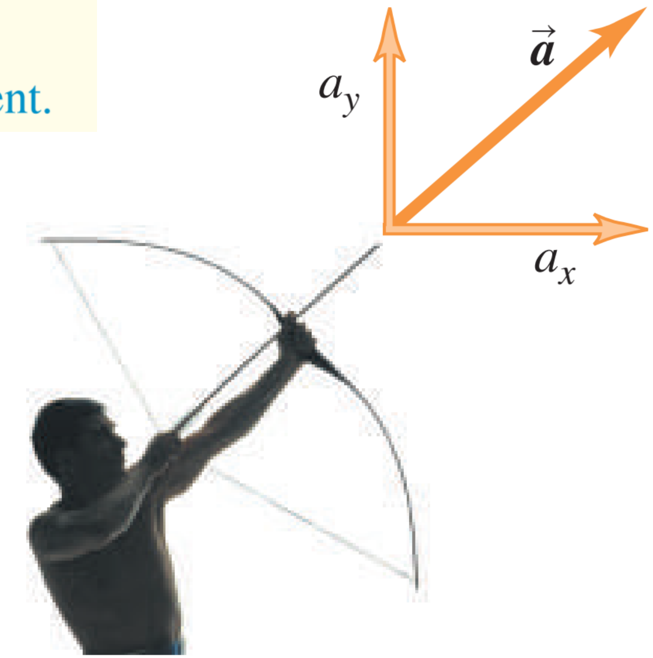
Each **component of** a particle's **instantaneous acceleration vector** ...

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt} \qquad a_z = \frac{dv_z}{dt}$$

... equals the instantaneous rate of change of its corresponding velocity component.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

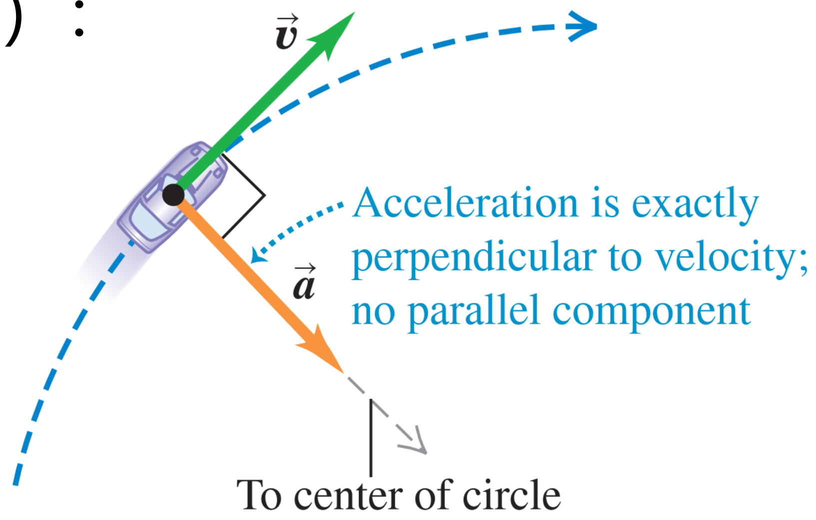
$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2}$$



看似“反常”的结论：任何曲线运动的质点都有加速度！

CAUTION **Any particle following a curved path is accelerating** When a particle is moving in a curved path, it always has nonzero acceleration, even when it moves with constant speed.

例：“匀速”圆周运动（uniform circular motion）：



其实速度在改变！！

为什么？　速度(velocity)是矢量，其方向在改变

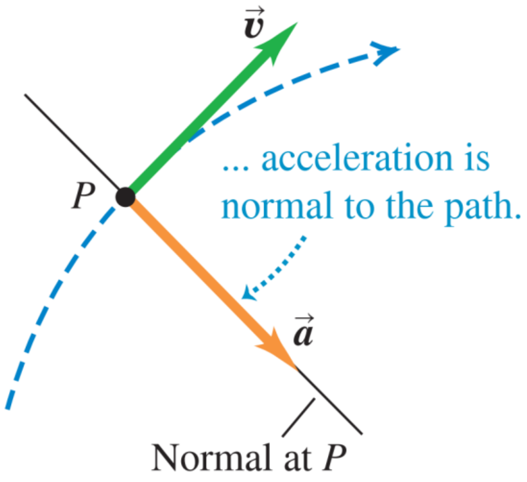
不变的是？　速率(speed)是标量，不可为负

课堂讨论：英文词义辨别：**平均、均匀**

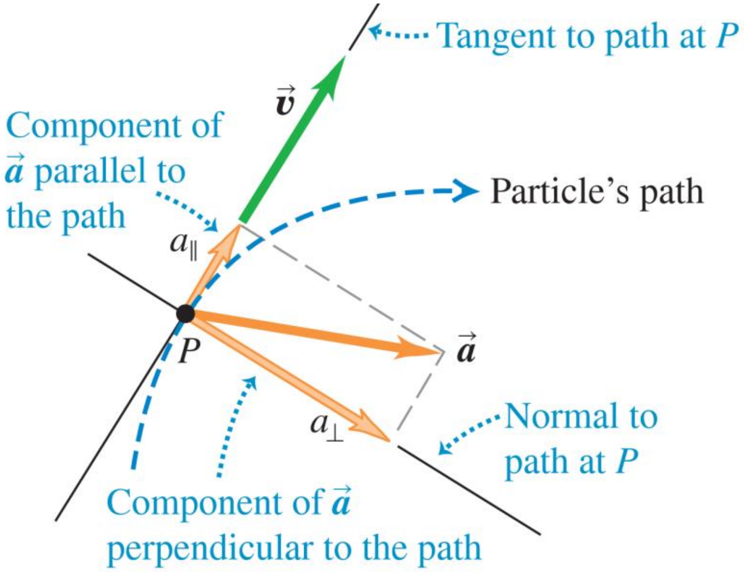
average、mean、even、uniform、homogenous、constant、…

匀速（率）曲线运动　　加速（率）曲线运动　　减速（率）曲线运动

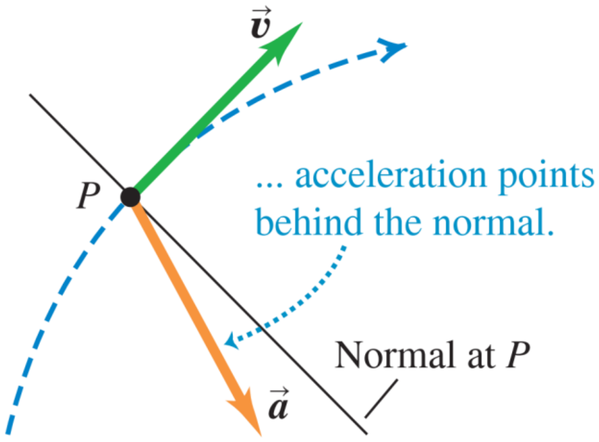
(a) When speed is constant along a curved path ...



(b) When speed is increasing along a curved path ...



(c) When speed is decreasing along a curved path ...



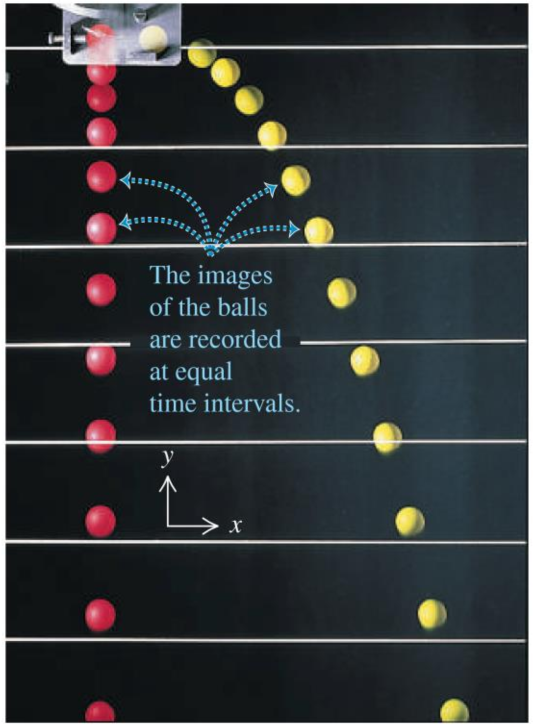
曲线运动的加速度可以分解为 **法向**和**切向**两个正交的坐标轴 上的分量

（4.1）重要的曲线运动 1：抛体运动

Projectile motion

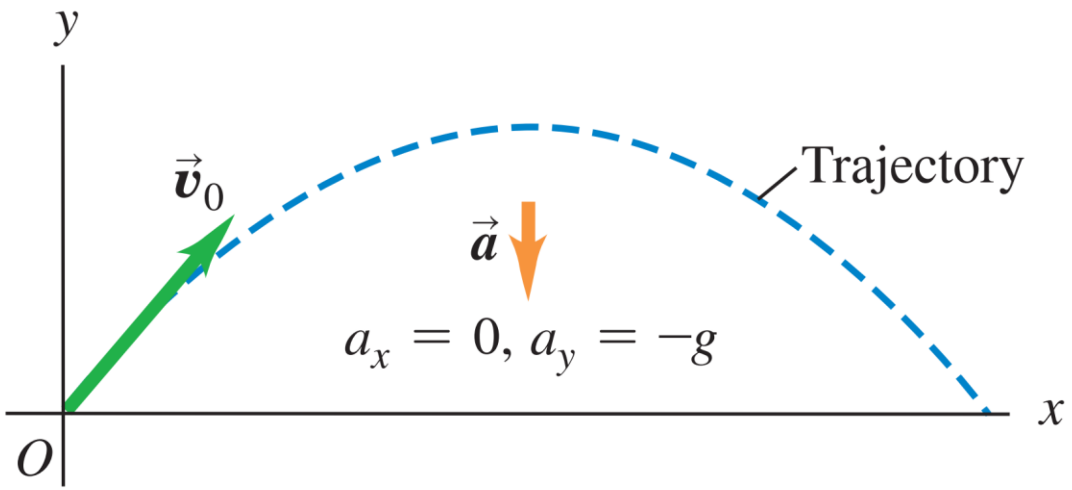
平抛

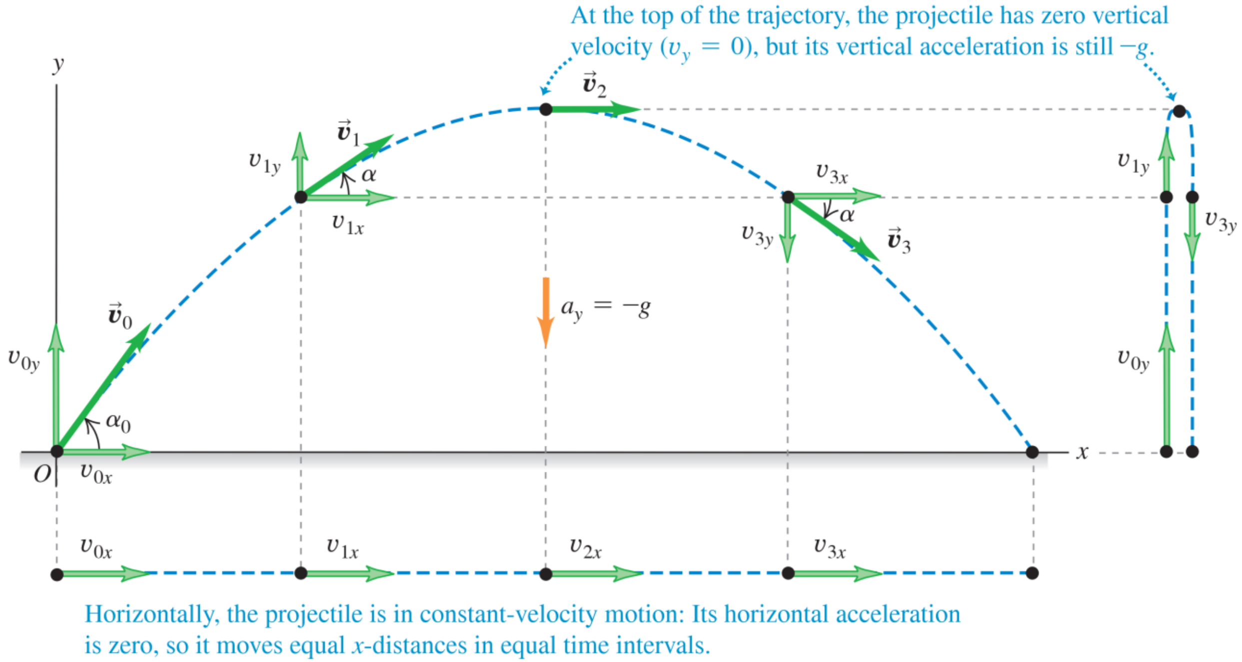
斜抛

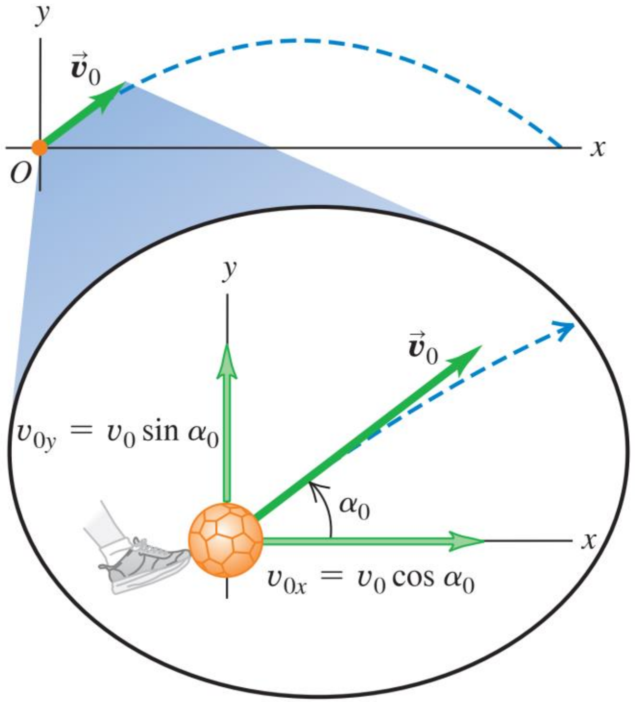


平抛是斜抛的一半

- A projectile moves in a vertical plane that contains the initial velocity vector $\vec{v}_0$.
- Its trajectory depends only on $\vec{v}_0$ and on the downward acceleration due to gravity.







上一节课的
一维匀加速运动方程：

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

抛体运动方程、运动轨迹：

Coordinates at time t of a **projectile** (positive y-direction is upward, and $x = y = 0$ at $t = 0$)

$$x = (v_0 \cos \alpha_0) t$$

$$y = (v_0 \sin \alpha_0) t - \frac{1}{2} g t^2$$

Speed at $t = 0$; Direction at $t = 0$; Time

Velocity components at time t of a **projectile** (positive y-direction is upward)

$$v_x = v_0 \cos \alpha_0$$

$$v_y = v_0 \sin \alpha_0 - g t$$

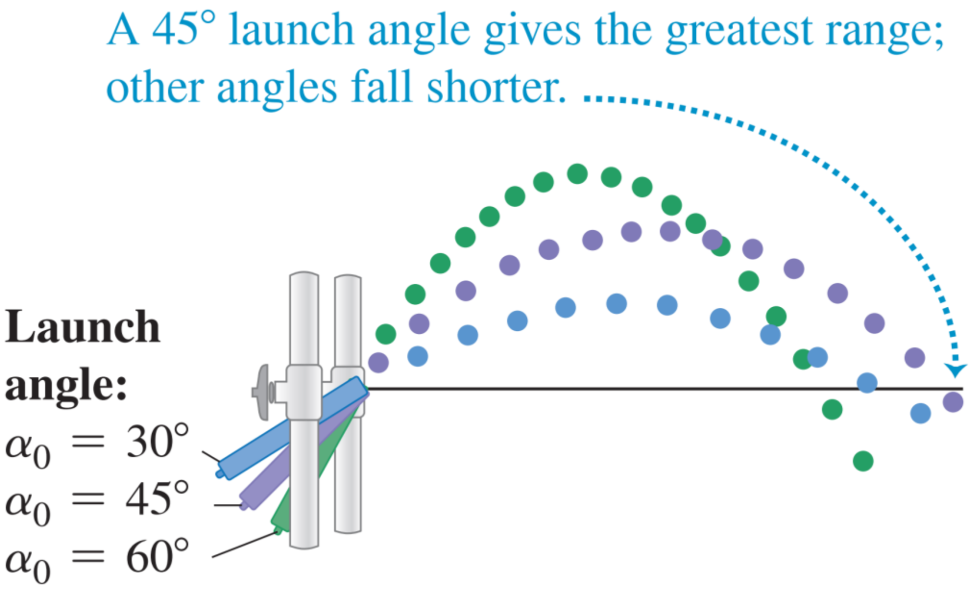
Speed at $t = 0$; Direction at $t = 0$; Time

Acceleration due to gravity: Note $g > 0$.

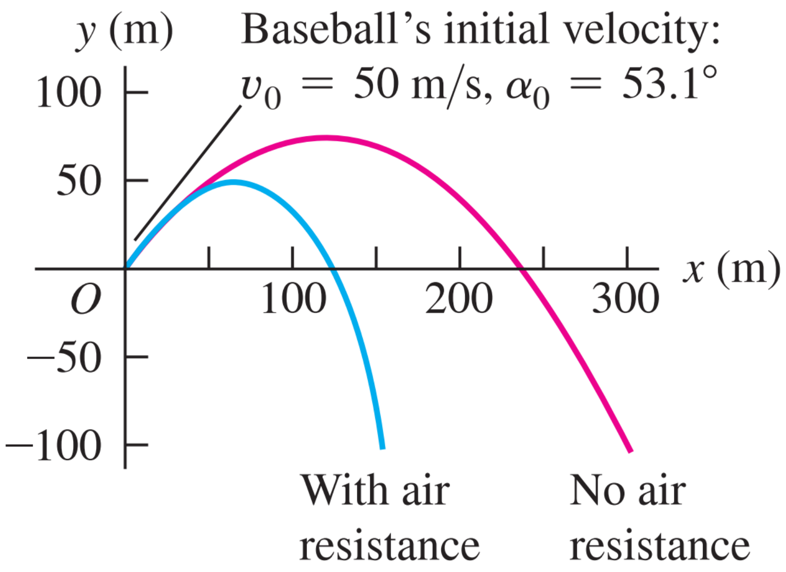
$$y = \underbrace{(\tan \alpha_0)}_{c} x - \underbrace{\frac{g}{2 v_0^2 \cos^2 \alpha_0}}_{b} x^2$$

$y = bx^2 + cx$ → 抛物线方程

Parabola equation



空气阻力是随速度变化的



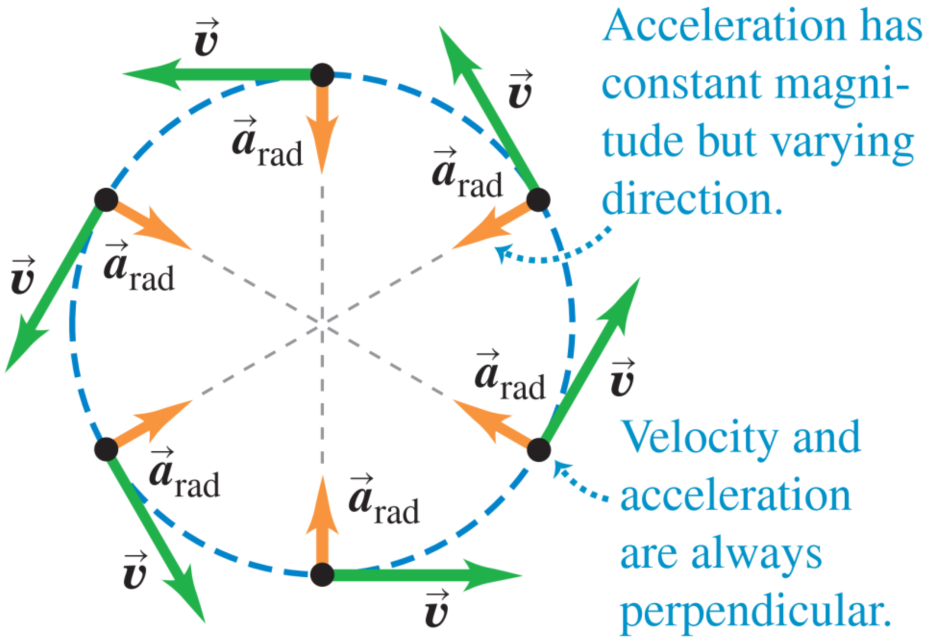
理想情况下，45度角抛射物体最远。

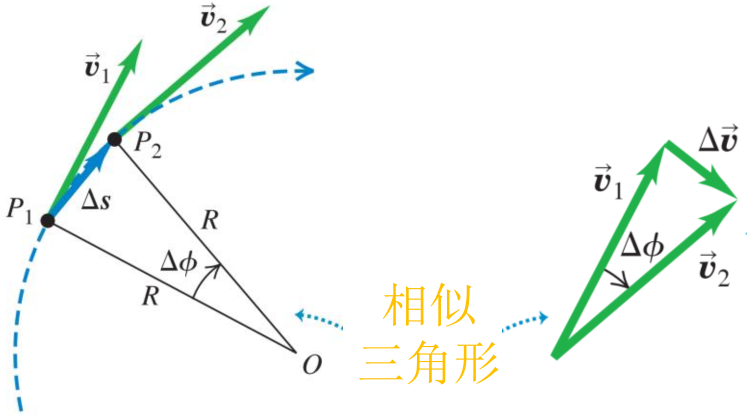
思考题：有空气阻力时，什么角度最远？

（4.2）重要的曲线运动 2：圆周运动

匀速圆周运动

速度大小保持不变、方向随时间变化





$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \quad \text{or} \quad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$

$$a_{av} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

$$a = \lim_{\Delta t\to 0}\frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}$$

the limit of $\Delta s/\Delta t$ is the speed v_1 at point P_1

加速度矢量 a_{rad}总是指向圆心，

又称为：径向加速度、向心加速度

Magnitude of acceleration of an object in **uniform circular motion**

$$a_{rad} = \frac{v^2}{R}$$

v: Speed of object

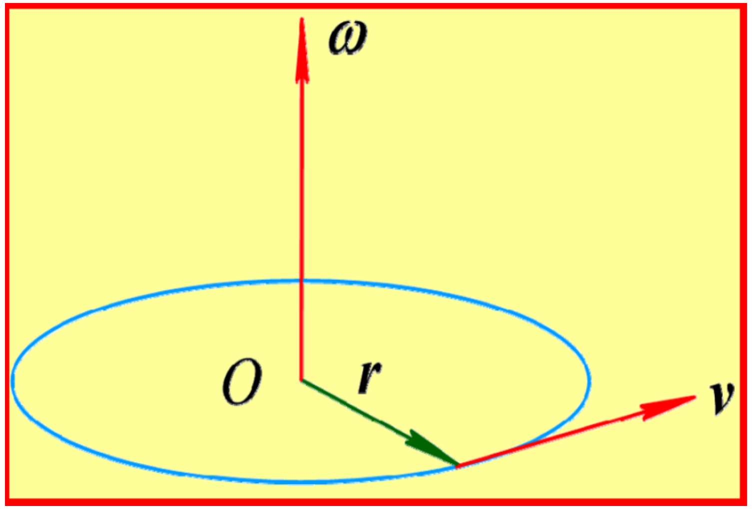
R: Radius of object's circular path

匀速圆周运动的周期 T、角速度 ω

定义角速度矢量，它的大小为单位时间内质点角位置的变化:

$$\omega = \lim_{\Delta t \to 0} \frac{|\Delta \varphi|}{\Delta t}$$

方向由右手法则确定。这样定义的量 $\vec{\omega}$ 称为角速度矢量。

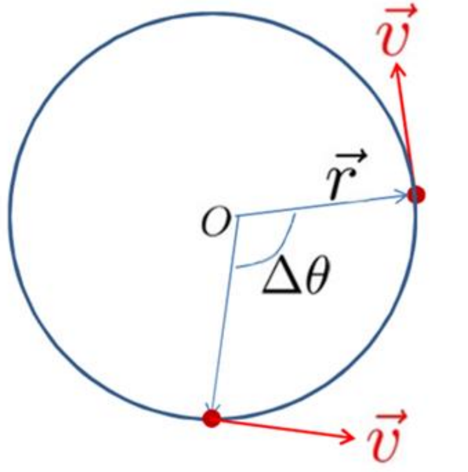


$$v = \frac{2\pi R}{T}$$

- 周期：运动一周所用的时间 $T = \dfrac{2\pi R}{v}$
- 角速度（大小）：

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{T} = \frac{v}{R}$$

$$\Rightarrow a = \omega^2 R$$

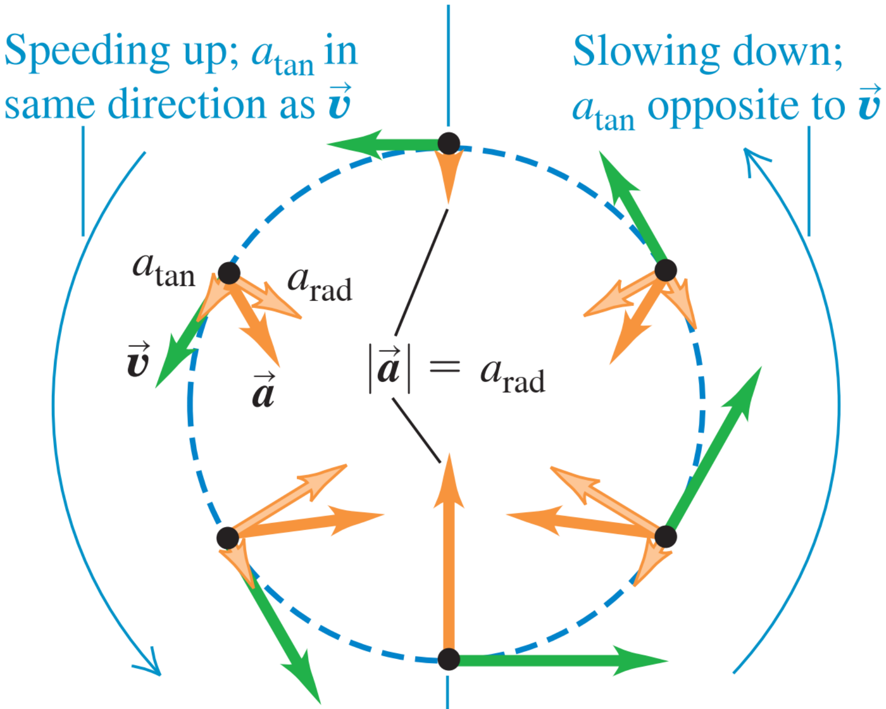


Magnitude of acceleration of an object in **uniform circular motion** → $a_{rad} = \dfrac{4\pi^2 R}{T^2}$ ← Radius of object's circular path; ← Period of motion

非匀速圆周运动

速度的方向、大小均随时间变化

(nonuniform circular motion)



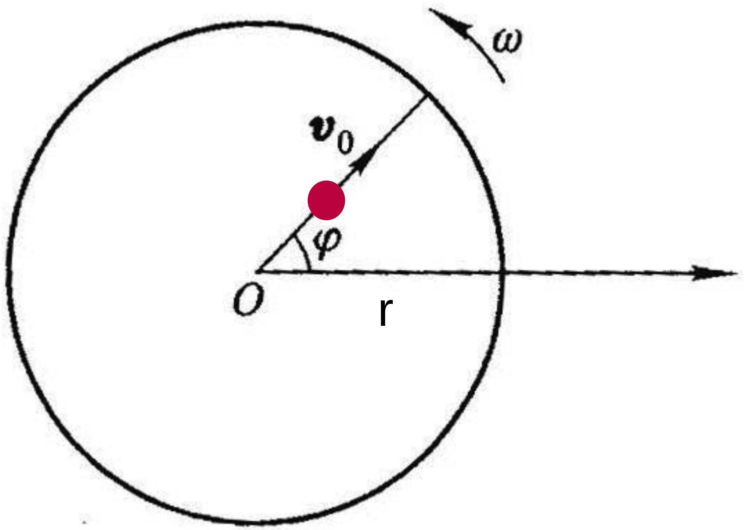
加速度矢量可以分解为径向和切向两个分量：

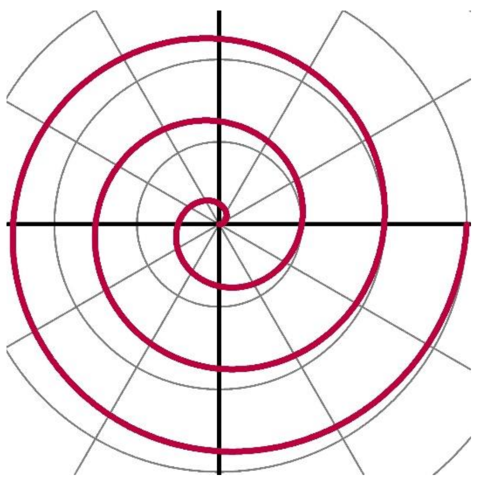
$$a_{rad} = \frac{v^2}{R} \quad \text{and} \quad a_{tan} = \frac{d|\vec{v}|}{dt}$$

课堂拓展讨论：

匀速圆周运动+匀速径向运动

一个匀速旋转的转盘上，一只蚂蚁从圆心往边缘以匀速率v_0沿径向直线爬行，其轨迹形成阿基米德螺线，请问这个过程中蚂蚁受到的加速度会改变吗？为什么？



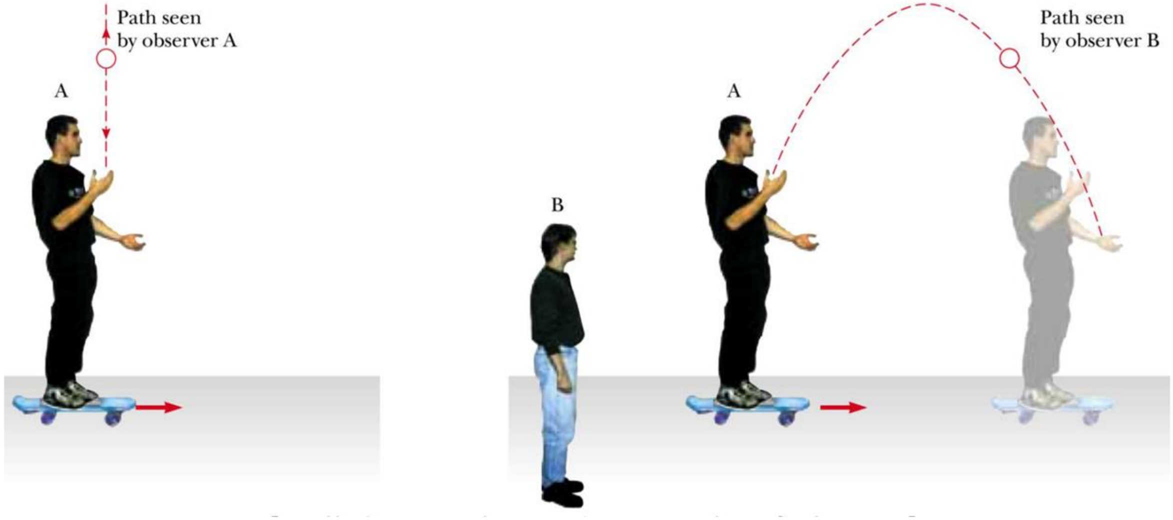


阿基米德螺线

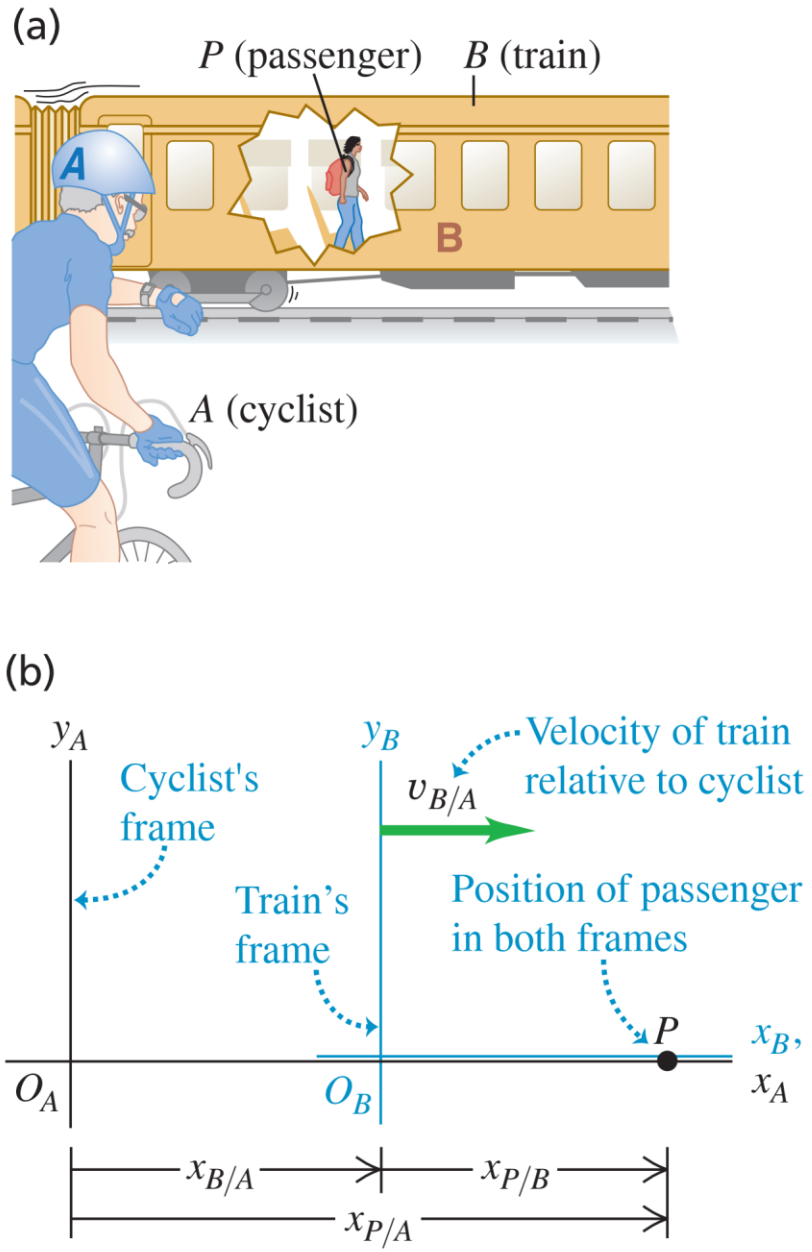
（4）参考系与相对运动

运动只能理解为物体的相对运动。在力学中，一般讲到运动，总是意味着相对于坐标系的运动。

——爱因斯坦



本节需要引入的概念：参考系、相对速度、*相对加速度



一维情况，考虑P点分别相对于两个参考系A、B的位置：

$$x_{P/A} = x_{P/B} + x_{B/A}$$

其中 $x_{B/A}$ 为参考系B相对于参考系A的相对位置。

微分之，得到：

$$\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt}$$

相对速度：

Relative velocity along a line:

$$v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x}$$

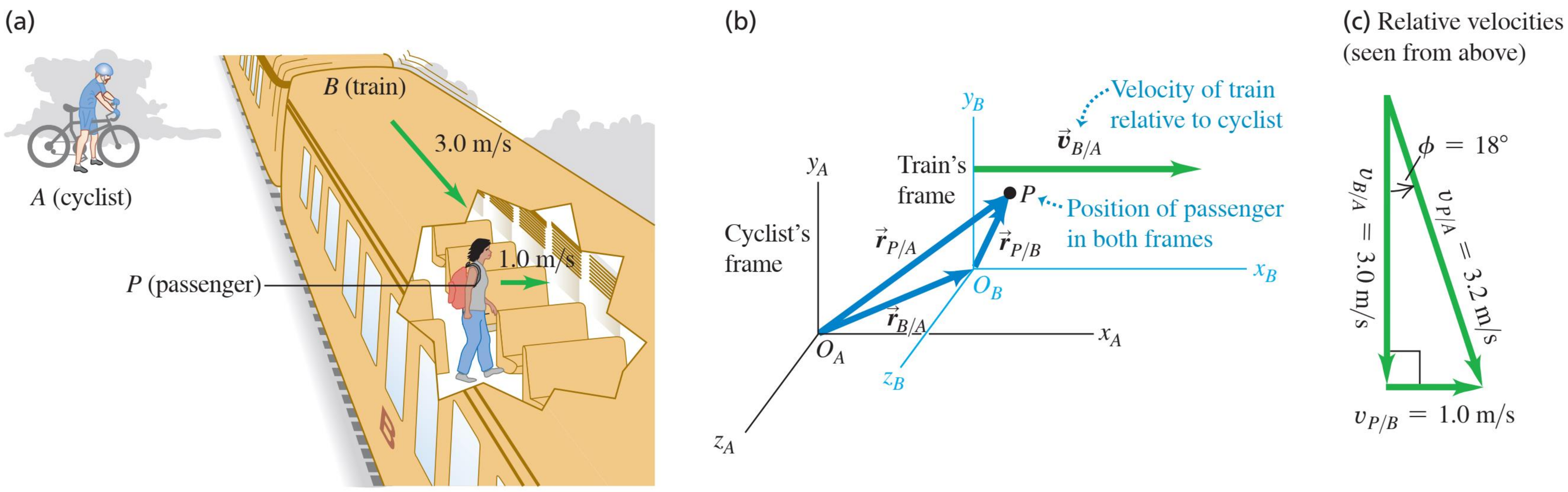
x-velocity of *P* relative to *A*

x-velocity of *P* relative to *B*

x-velocity of *B* relative to *A*

注意： $v_{A/B\text{-}x} = -v_{B/A\text{-}x}$

三维空间情况，考虑P点分别相对于两个参考系A、B的位置：



$$\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}$$

，同理，微分之得到：

相对速度：

Relative velocity in space:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

Velocity of *P* relative to *A*

Velocity of *P* relative to *B*

Velocity of *B* relative to *A*

伽利略速度变换 *Galilean velocity transformation* ——成立前提：v << c (光速)

本章内容概览：

（1）基本概念：速度，瞬时速度

（2）微积分初步

（3）基本概念：
位置、速度、加速度 矢量

（4.1）抛体运动

（4.2）圆周运动

（5）相对运动

（6）拓展：其他坐标系

Position, velocity, and acceleration vectors: The position vector $\vec{r}$ of a point P in space is the vector from the origin to P. Its components are the coordinates x, y, and z.

The average velocity vector $\vec{v}_{av}$ during the time interval Δt is the displacement $\Delta\vec{r}$ (the change in position vector $\vec{r}$) divided by Δt. The instantaneous velocity vector $\vec{v}$ is the time derivative of $\vec{r}$, and its components are the time derivatives of x, y, and z. The instantaneous speed is the magnitude of $\vec{v}$. The velocity $\vec{v}$ of a particle is always tangent to the particle's path. (See Example 3.1.)

The average acceleration vector $\vec{a}_{av}$ during the time interval Δt equals $\Delta\vec{v}$ (the change in velocity vector $\vec{v}$) divided by Δt. The instantaneous acceleration vector $\vec{a}$ is the time derivative of $\vec{v}$, and its components are the time derivatives of v_x, v_y, and v_z. (See Example 3.2.)

The component of acceleration parallel to the direction of the instantaneous velocity affects the speed, while the component of $\vec{a}$ perpendicular to $\vec{v}$ affects the direction of motion. (See Examples 3.3 and 3.4.)

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \quad (3.1)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \quad (3.2)$$

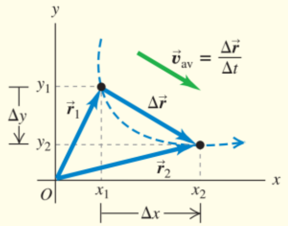
$$\vec{v} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \quad (3.3)$$

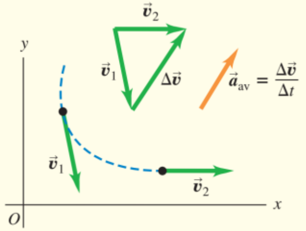
$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt} \quad (3.4)$$

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \quad (3.8)$$

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad (3.9)$$

$$a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt} \quad a_z = \frac{dv_z}{dt} \quad (3.10)$$





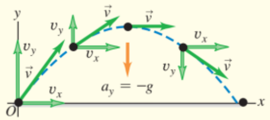
Projectile motion: In projectile motion with no air resistance, $a_x = 0$ and $a_y = -g$. The coordinates and velocity components are simple functions of time, and the shape of the path is always a parabola. We usually choose the origin to be at the initial position of the projectile. (See Examples 3.5–3.10.)

$$x = (v_0\cos\alpha_0)t \quad (3.19)$$

$$y = (v_0\sin\alpha_0)t - \tfrac{1}{2}gt^2 \quad (3.20)$$

$$v_x = v_0\cos\alpha_0 \quad (3.21)$$

$$v_y = v_0\sin\alpha_0 - gt \quad (3.22)$$

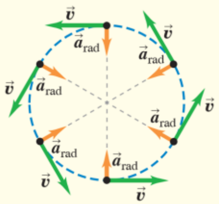


Uniform and nonuniform circular motion: When a particle moves in a circular path of radius R with constant speed v (uniform circular motion), its acceleration $\vec{a}$ is directed toward the center of the circle and perpendicular to $\vec{v}$. The magnitude a_{rad} of this radial acceleration can be expressed in terms of v and R or in terms of R and the period T (the time for one revolution), where $v = 2\pi R/T$. (See Examples 3.11 and 3.12.)

If the speed is not constant in circular motion (nonuniform circular motion), there is still a radial component of $\vec{a}$ given by Eq. (3.27) or (3.29), but there is also a component of $\vec{a}$ parallel (tangential) to the path. This tangential component is equal to the rate of change of speed, dv/dt.

$$a_{rad} = \frac{v^2}{R} \quad (3.27)$$

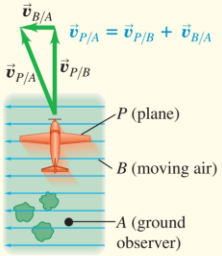
$$a_{rad} = \frac{4\pi^2 R}{T^2} \quad (3.29)$$



Relative velocity: When an object P moves relative to an object (or reference frame) B, and B moves relative to an object (or reference frame) A, we denote the velocity of P relative to B by $\vec{v}_{P/B}$, the velocity of P relative to A by $\vec{v}_{P/A}$, and the velocity of B relative to A by $\vec{v}_{B/A}$. If these velocities are all along the same line, their components along that line are related by Eq. (3.32). More generally, these velocities are related by Eq. (3.35). (See Examples 3.13–3.15.)

$$v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x} \quad (3.32)$$
(relative velocity along a line)

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \quad (3.35)$$
(relative velocity in space)



基本概念

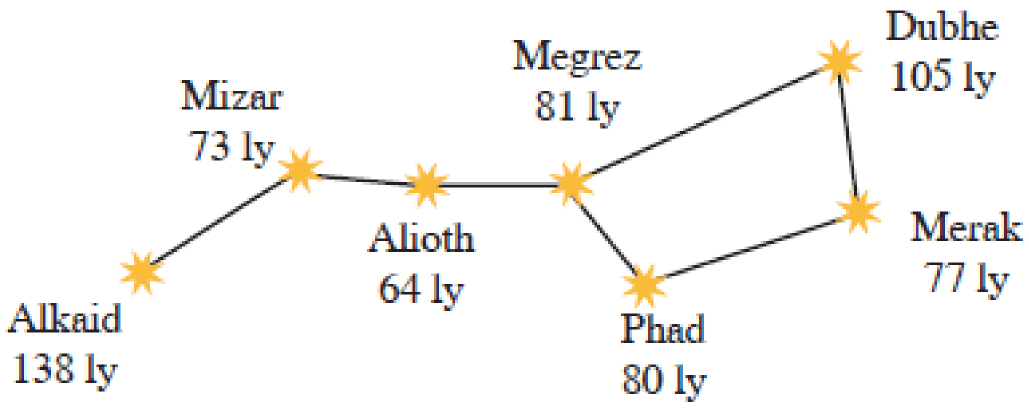
案例分析

单体 → 二体

Example #1

1.91 ••• **Navigating in the Big Dipper.** All of the stars of the Big Dipper (part of the constellation Ursa Major) may appear to be the same distance from the earth, but in fact they are very far from each other. **Figure P1.91** shows the distances from the earth to each of these stars. The distances are given in light-years (ly), the distance that light travels in one year. One light-year equals 9.461×10^{15} m. (a) Alkaid and Merak are 25.6° apart in the earth's sky. In a diagram, show the relative positions of Alkaid, Merak, and our sun. Find the distance in light-years from Alkaid to Merak. (b) To an inhabitant of a planet orbiting Merak, how many degrees apart in the sky would Alkaid and our sun be?

Figure P1.91



Example #2

MCAT-STYLE PASSAGE PROBLEMS

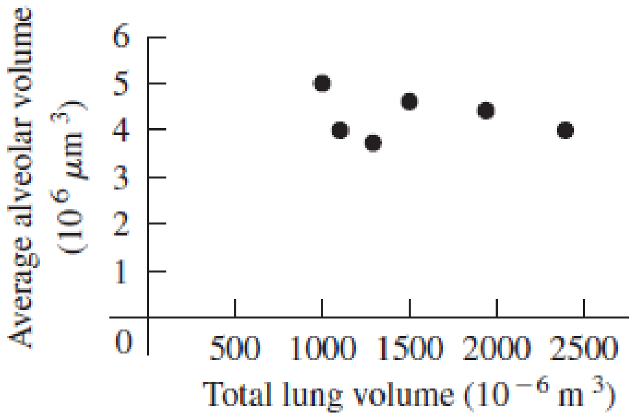
BIO Calculating Lung Volume in Humans. In humans, oxygen and carbon dioxide are exchanged in the blood within many small sacs called alveoli in the lungs. Alveoli provide a large surface area for gas exchange. Recent careful measurements show that the total number of alveoli in a typical pair of lungs is about 480×10^6 and that the average volume of a single alveolus is $4.2 \times 10^6\ \mu m^3$. (The volume of a sphere is $V = \frac{4}{3}\pi r^3$, and the area of a sphere is $A = 4\pi r^2$.)

1.92 What is total volume of the gas-exchanging region of the lungs? (a) $2000\ \mu m^3$; (b) $2\ m^3$; (c) 2.0 L; (d) 120 L.

1.93 If we assume that alveoli are spherical, what is the diameter of a typical alveolus? (a) 0.20 mm; (b) 2 mm; (c) 20 mm; (d) 200 mm.

1.94 Individuals vary considerably in total lung volume. **Figure P1.94** shows the results of measuring the total lung volume and average alveolar volume of six individuals. From these data, what can you infer about the relationship among alveolar size, total lung volume, and number of alveoli per individual? As the total volume of the lungs increases, (a) the number and volume of individual alveoli increase; (b) the number of alveoli increases and the volume of individual alveoli decreases; (c) the volume of the individual alveoli remains constant and the number of alveoli increases; (d) both the number of alveoli and the volume of individual alveoli remain constant.

Figure P1.94



Example #3

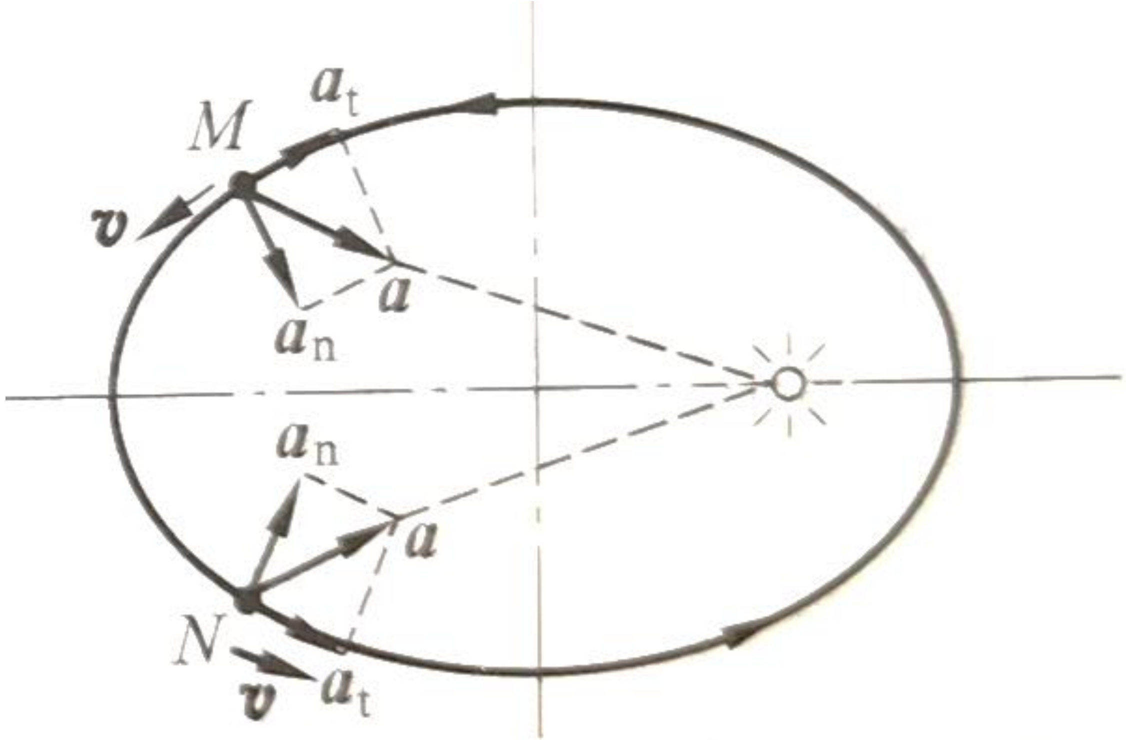
2.88 ••• Catching the Bus. A student is running at her top speed of 5.0 m/s to catch a bus, which is stopped at the bus stop. When the student is still 40.0 m from the bus, it starts to pull away, moving with a constant acceleration of 0.170 m/s^2. (a) For how much time and what distance does the student have to run at 5.0 m/s before she overtakes the bus? (b) When she reaches the bus, how fast is the bus traveling? (c) Sketch an x-t graph for both the student and the bus. Take $x = 0$ at the initial position of the student. (d) The equations you used in part (a) to find the time have a second solution, corresponding to a later time for which the student and bus are again at the same place if they continue their specified motions. Explain the significance of this second solution. How fast is the bus traveling at this point? (e) If the student's top speed is 3.5 m/s, will she catch the bus? (f) What is the *minimum* speed the student must have to just catch up with the bus? For what time and what distance does she have to run in that case?

Example #4

3.81 ••• **CP** A rocket designed to place small payloads into orbit is carried to an altitude of 12.0 km above sea level by a converted airliner. When the airliner is flying in a straight line at a constant speed of 850 km/h, the rocket is dropped. After the drop, the airliner maintains the same altitude and speed and continues to fly in a straight line. The rocket falls for a brief time, after which its rocket motor turns on. Once that motor is on, the combined effects of thrust and gravity give the rocket a constant acceleration of magnitude $3.00g$ directed at an angle of 30.0° above the horizontal. For safety, the rocket should be at least 1.00 km in front of the airliner when it climbs through the airliner's altitude. Your job is to determine the minimum time that the rocket must fall before its engine starts. Ignore air resistance. Your answer should include (i) a diagram showing the flight paths of both the rocket and the airliner, labeled at several points with vectors for their velocities and accelerations; (ii) an x-t graph showing the motions of both the rocket and the airliner; and (iii) a y-t graph showing the motions of both the rocket and the airliner. In the diagram and the graphs, indicate when the rocket is dropped, when the rocket motor turns on, and when the rocket climbs through the altitude of the airliner.

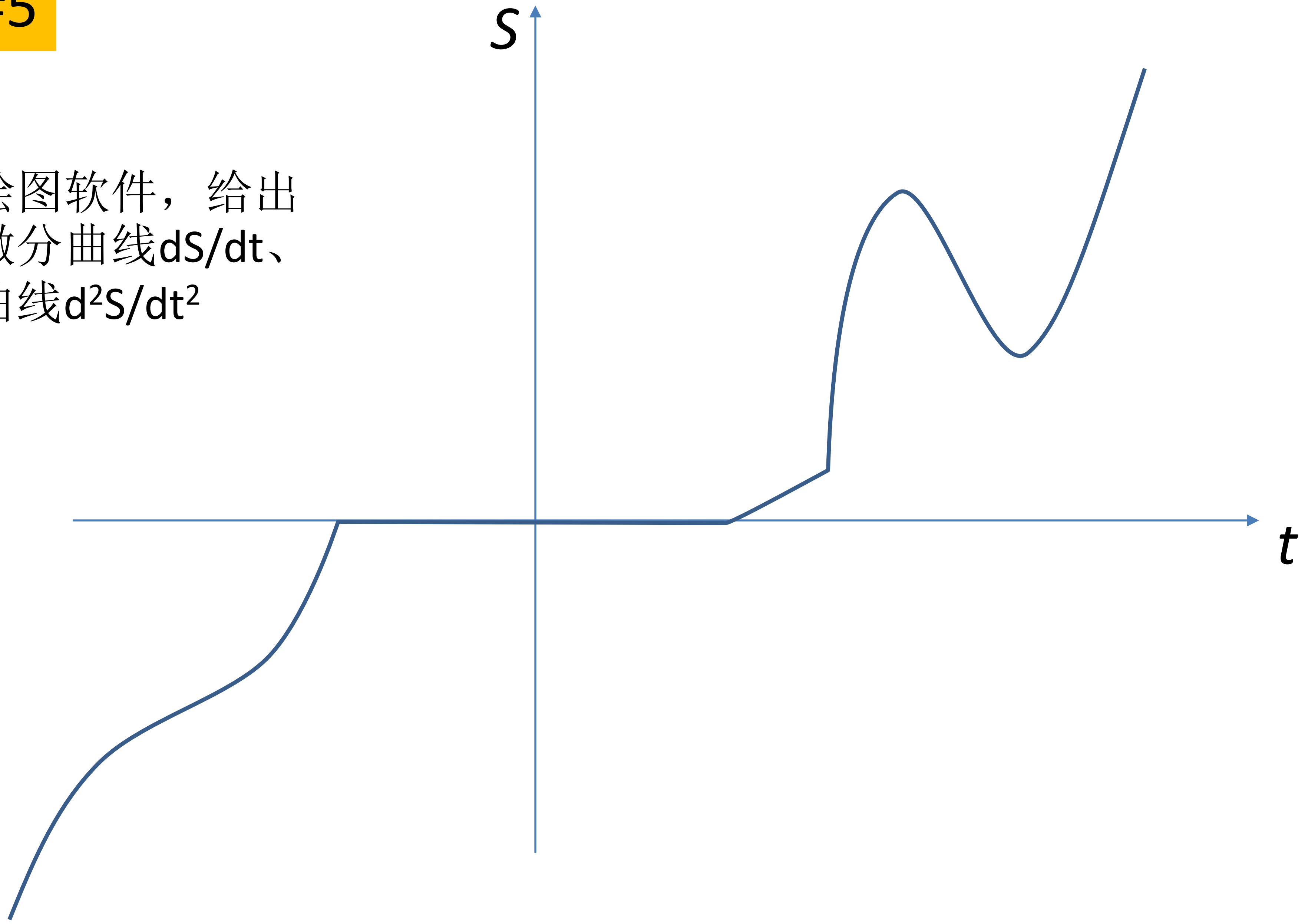
Example #5

根据开普勒第一定律，行星轨道为椭圆（图1.1）。已知任一时刻行星的加速度方向都指向椭圆的一个焦点（太阳所在处）。分析行星在通过图中M，N两位置时，它的速率分别应正在增大还是正在减小？



Example #5

使用科学绘图软件，给出该曲线的微分曲线dS/dt、二次微分曲线d²S/dt²

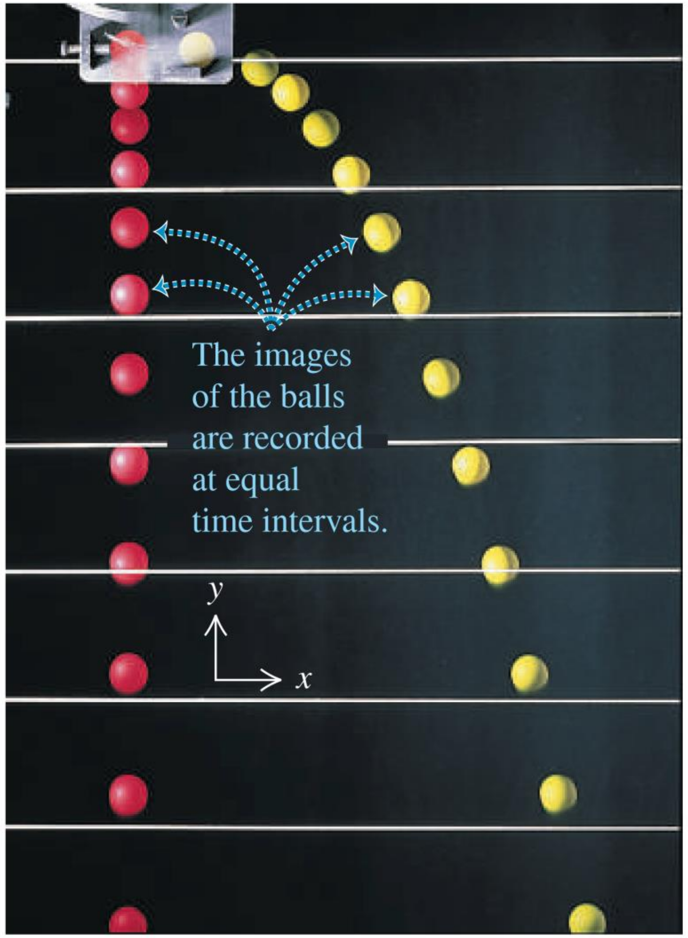


Example #6

平抛运动：

平抛运动从侧面看，是平抛运动，从正面或后面看，则是自由落体。

请问，从45度角看，是什么运动？请给出运动方程。



- At any time the two balls have different x-coordinates and x-velocities but the same y-coordinate, y-velocity, and y-acceleration.
- The horizontal motion of the yellow ball has no effect on its vertical motion.

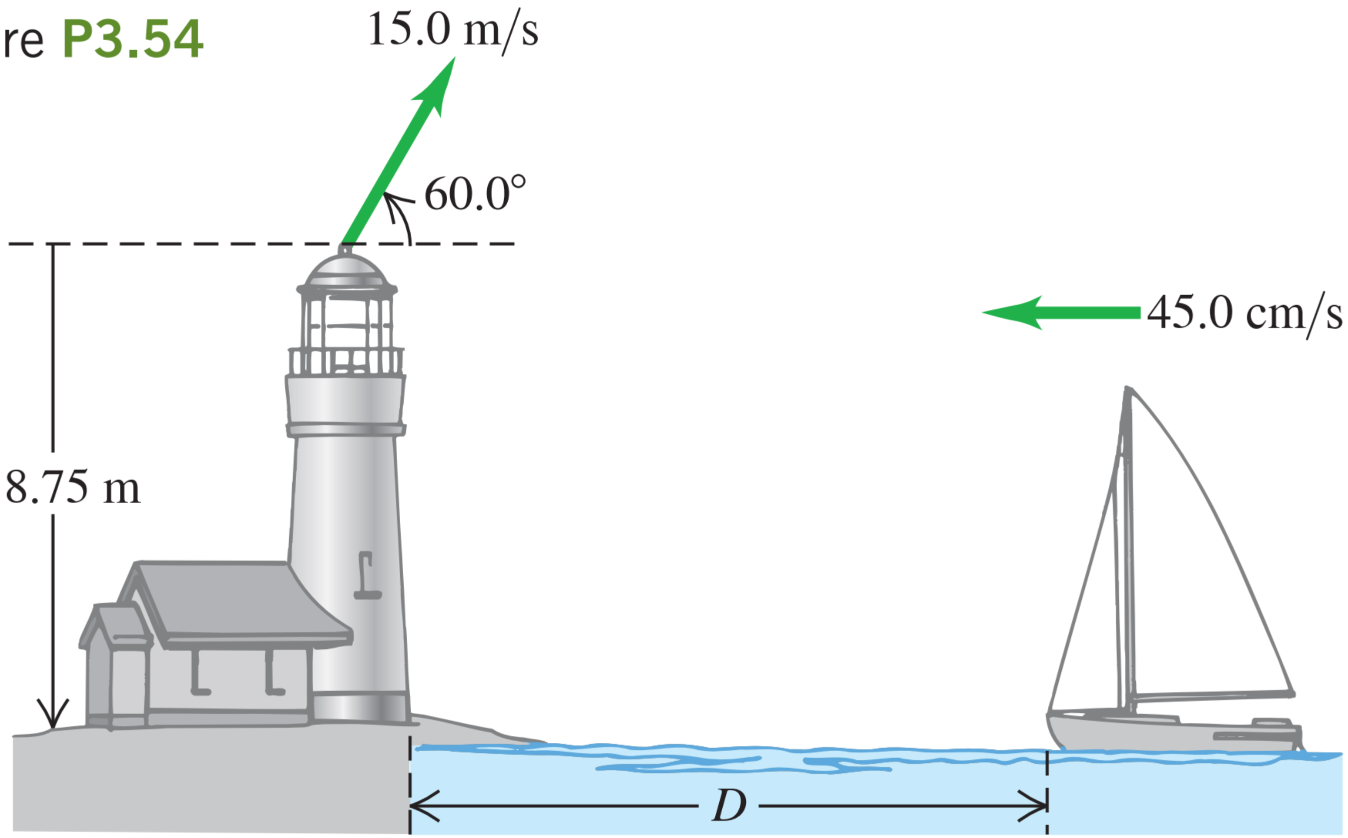
Homework #1

2.87 ••• In the vertical jump, an athlete starts from a crouch and jumps upward as high as possible. Even the best athletes spend little more than 1.00 s in the air (their "hang time"). Treat the athlete as a particle and let y_{max} be his maximum height above the floor. To explain why he seems to hang in the air, calculate the ratio of the time he is above $y_{max}/2$ to the time it takes him to go from the floor to that height. Ignore air resistance.

Homework #2

3.54 ••• An important piece of landing equipment must be thrown to a ship, which is moving at 45.0 cm/s, before the ship can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck (**Fig. P3.54**). For this equipment to land at the front of the ship, at what distance D from the dock should the ship be when the equipment is thrown? Ignore air resistance.

Figure **P3.54**



Homework #3

3.79 ••• CALC A projectile thrown from a point P moves in such a way that its distance from P is always increasing. Find the maximum angle above the horizontal with which the projectile could have been thrown. Ignore air resistance.

Homework #4

一张致密光盘(CD)音轨区域的内半径R_1=2.2cm,外半径为R_2=5.6cm,径向音轨密度N=650条/mm。在CD唱机内,光盘每转一圈,激光头沿径向向外移动一条音轨,激光束相对光盘是以v=1.3m/s的恒定线速度运动的。

(1)这张光盘的全部放音时间是多少?

(2)激光束到达离盘心r=5.0cm处时,光盘转动的角速度和角加速度各是多少?