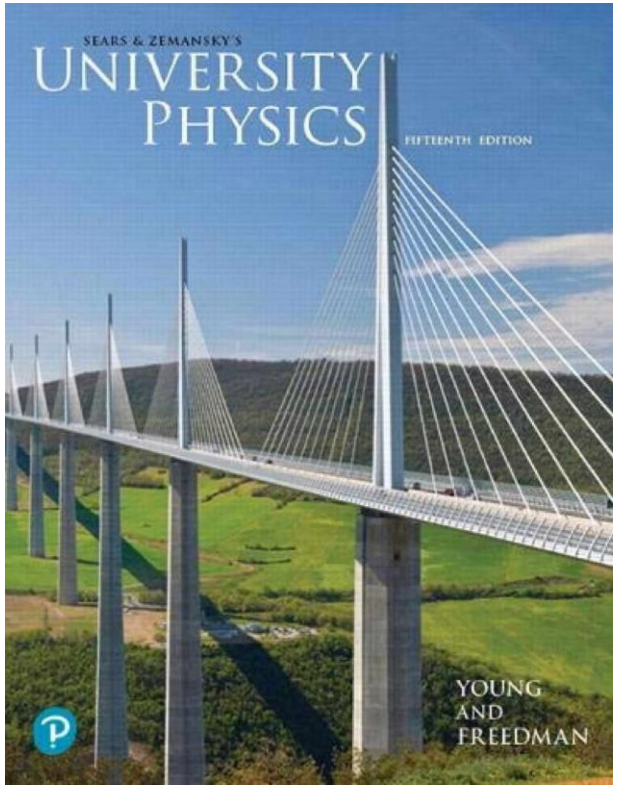
普通物理I PHYS1181 第13讲

振动
Oscillation



本节课（振动）：

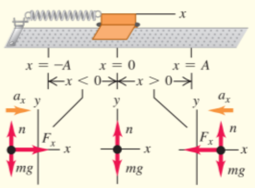
A. 振动的基本概念
B. 简谐振动
C. 简谐振动的能量
D. 扭摆
E. 单摆
F. 复摆

CHAPTER 14 SUMMARY

Periodic motion: Periodic motion is motion that repeats itself in a definite cycle. It occurs whenever an object has a stable equilibrium position and a restoring force that acts when the object is displaced from equilibrium. Period T is the time for one cycle. Frequency f is the number of cycles per unit time. Angular frequency ω is 2π times the frequency. (See Example 14.1.)

$$f = \frac{1}{T} \qquad T = \frac{1}{f} \tag{14.1}$$

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{14.2}$$



Simple harmonic motion: If the restoring force F_x in periodic motion is directly proportional to the displacement x, the motion is called simple harmonic motion (SHM). In many cases this condition is satisfied if the displacement from equilibrium is small. The angular frequency, frequency, and period in SHM do not depend on the amplitude but on only the mass m and force constant k. The displacement, velocity, and acceleration in SHM are sinusoidal functions of time; the amplitude A and phase angle ϕ of the oscillation are determined by the initial displacement and velocity of the object. (See Examples 14.2, 14.3, 14.6, and 14.7.)

$$F_x = -kx \tag{14.3}$$

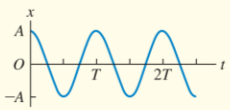
$$a_x = \frac{F_x}{m} = -\frac{k}{m}x \tag{14.4}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{14.10}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{14.11}$$

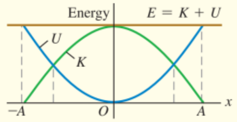
$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \tag{14.12}$$

$$x = A\cos(\omega t + \phi) \tag{14.13}$$



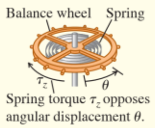
Energy in simple harmonic motion: Energy is conserved in SHM. The total energy can be expressed in terms of the force constant k and amplitude A. (See Examples 14.4 and 14.5.)

$$E = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \text{constant} \tag{14.21}$$



Angular simple harmonic motion: In angular SHM, the frequency and angular frequency are related to the moment of inertia I and the torsion constant κ.

$$\omega = \sqrt{\frac{\kappa}{I}} \quad \text{and} \quad f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}} \tag{14.24}$$

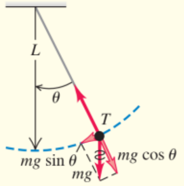


Simple pendulum: A simple pendulum consists of a point mass m at the end of a massless string of length L. Its motion is approximately simple harmonic for sufficiently small amplitude; the angular frequency, frequency, and period then depend on only g and L, not on the mass or amplitude. (See Example 14.8.)

$$\omega = \sqrt{\frac{g}{L}} \tag{14.32}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \tag{14.33}$$

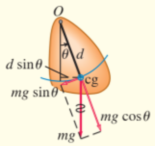
$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}} \tag{14.34}$$



Physical pendulum: A physical pendulum is any object suspended from an axis of rotation. The angular frequency and period for small-amplitude oscillations are independent of amplitude but depend on the mass m, distance d from the axis of rotation to the center of gravity, and moment of inertia I about the axis. (See Examples 14.9 and 14.10.)

$$\omega = \sqrt{\frac{mgd}{I}} \tag{14.38}$$

$$T = 2\pi\sqrt{\frac{I}{mgd}} \tag{14.39}$$



本节课（振动）：

G. 阻尼振动

Q值

H. 受迫振动、共振

I. 简谐振动的合成

同方向同频率

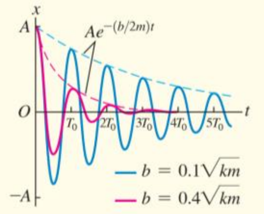
同方向不同频率、拍

垂直方向、李萨如图形

Damped oscillations: When a force $F_x = -bv_x$ is added to a simple harmonic oscillator, the motion is called a damped oscillation. If $b < 2\sqrt{km}$ (called underdamping), the system oscillates with a decaying amplitude and an angular frequency ω' that is lower than it would be without damping. If $b = 2\sqrt{km}$ (called critical damping) or $b > 2\sqrt{km}$ (called overdamping), when the system is displaced it returns to equilibrium without oscillating.

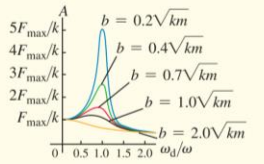
$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi) \quad (14.42)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \quad (14.43)$$



Forced oscillations and resonance: When a sinusoidally varying driving force is added to a damped harmonic oscillator, the resulting motion is called a forced oscillation or driven oscillation. The amplitude is a function of the driving frequency ω_d and reaches a peak at a driving frequency close to the natural frequency of the system. This behavior is called resonance.

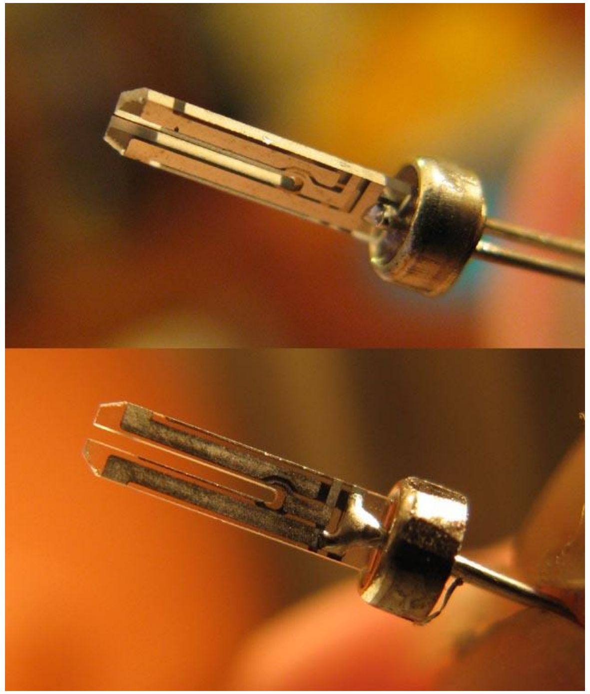
$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}} \quad (14.46)$$



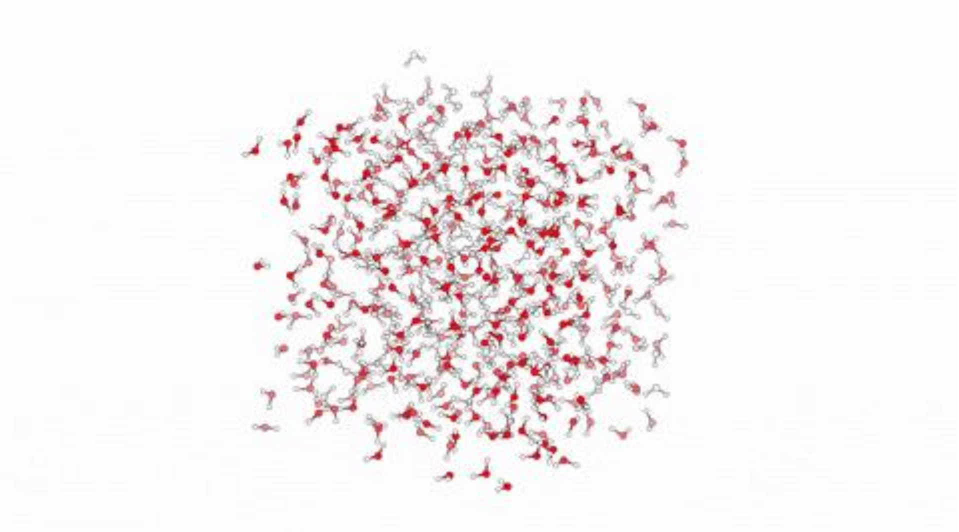
振动是自然界极为普遍的现象。

石英手表中的晶振

实际上就是一个石英做的
小音叉 (tuning fork)



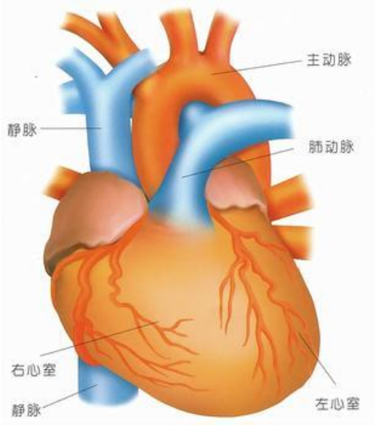
只要不是绝对零度，物质中所有的分子、所有的原子就都在不断的振动



水团簇的分子动力学模拟

人体中的振动

心脏的搏动
声带、耳膜的振动



A. 振动（周期运动）

A1：什么是振动?

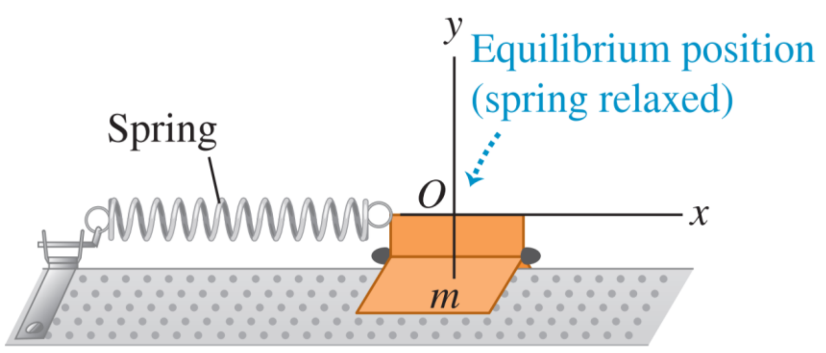
周期性的运动 periodic motion

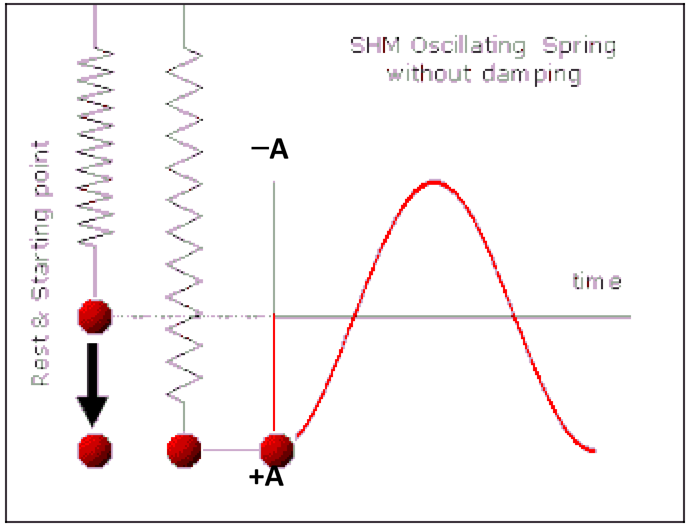
振：vibration（振动），oscillation（振荡）

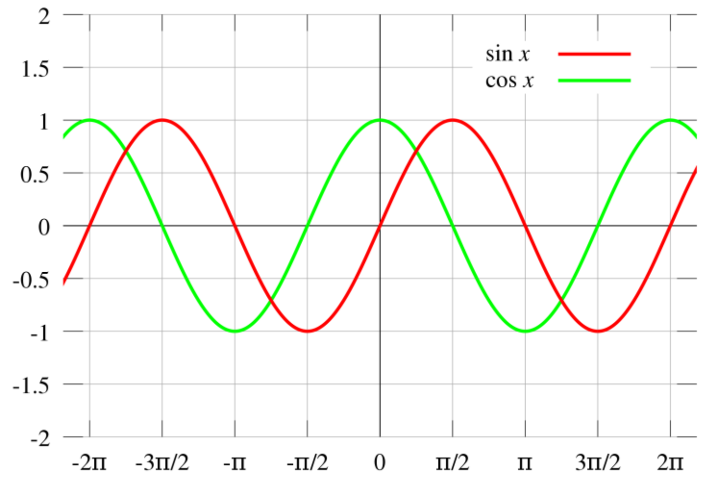
振动 ≠ 震动　　地震
手机震动
古龙小说：虎躯一震

振荡 ≠ 震荡　　股市震荡

震：quake, shake







A2：振动的几个基本概念

幅度 A

周期 T

频率 f

角频率 ω

赫兹

$$1\ \text{hertz} = 1\ \text{Hz} = 1\ \text{cycle/s} = 1\ \text{s}^{-1}$$

In periodic motion **frequency and period** are reciprocals of each other.

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

Period

Frequency

Angular frequency related to frequency and period

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Frequency

Period

B. 简谐振动（simple harmonic oscillation）

B1：简谐运动（SHM）：弹性力 *F* 与偏离平衡位置的位移 *x* 成正比

理想弹簧的弹性力，满足线性的胡克定律：

k ——弹簧的弹性系数（劲度系数）

$$F_x = -kx$$

- **Restoring force** exerted by an **ideal spring** → F_x
- x-component of force → F_x
- Displacement → x
- Force constant of spring → k

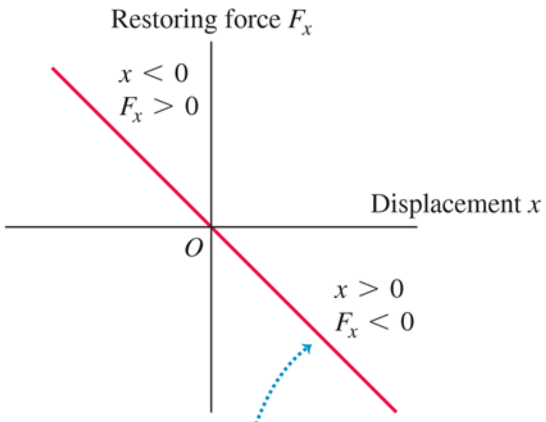
质量为 *m* 物体做简谐振动的加速度方程：

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

- **Equation for simple harmonic motion**
- x-component of acceleration → a_x
- Second derivative of displacement → $\frac{d^2x}{dt^2}$
- Force constant of restoring force → k
- Displacement → x
- Mass of object → m

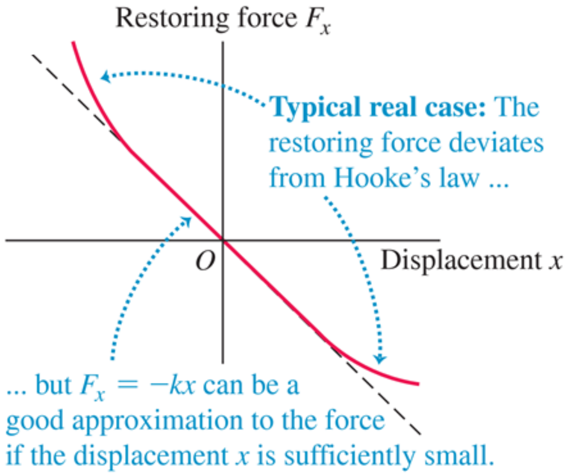
- 简谐振动中，周期与频率不依赖于幅度 A.

理想弹簧



The restoring force exerted by an idealized spring is directly proportional to the displacement (Hooke's law, $F_x = -kx$): the graph of F_x versus x is a straight line.

实际弹簧



B2：简谐振动的描述

由简谐运动的加速度方程：$F_x = ma = m\dfrac{d^2x}{dt^2} = -kx$

定义**角频率** ω：

Angular frequency for simple harmonic motion → $\omega = \sqrt{\dfrac{k}{m}}$ ← Force constant of restoring force (k); Mass of object (m)

$$\omega^2 = \frac{k}{m}$$

可得：

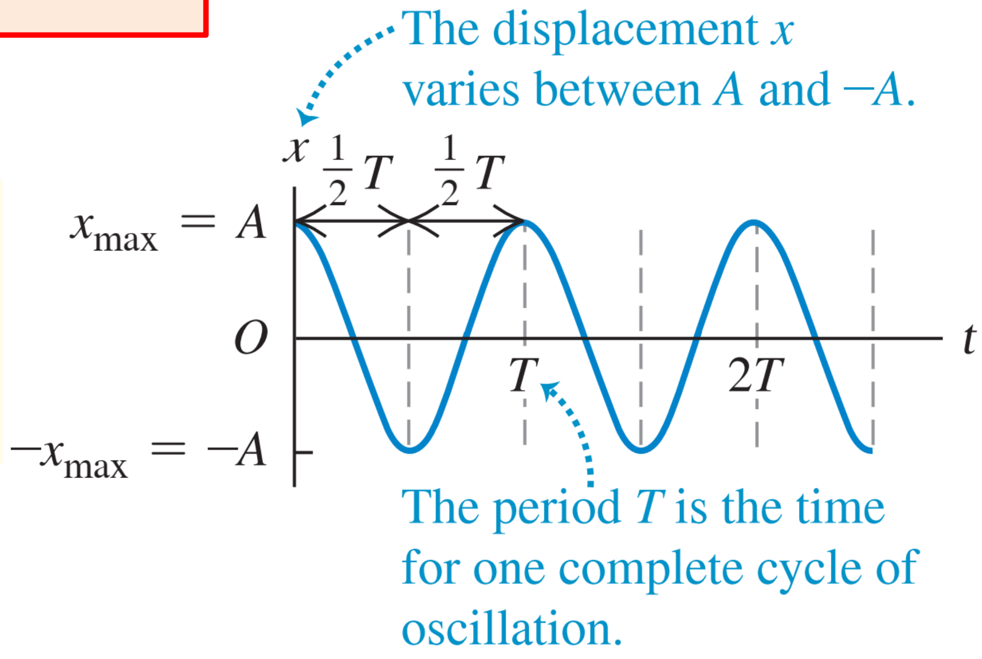
简谐振动的特征方程

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

解此微分方程，可得**简谐运动方程**：

Displacement in simple harmonic motion as a function of time → $x = A\cos(\omega t + \phi)$ (Amplitude: A; Time: t; Phase angle: ϕ)

Angular frequency $= \sqrt{k/m}$



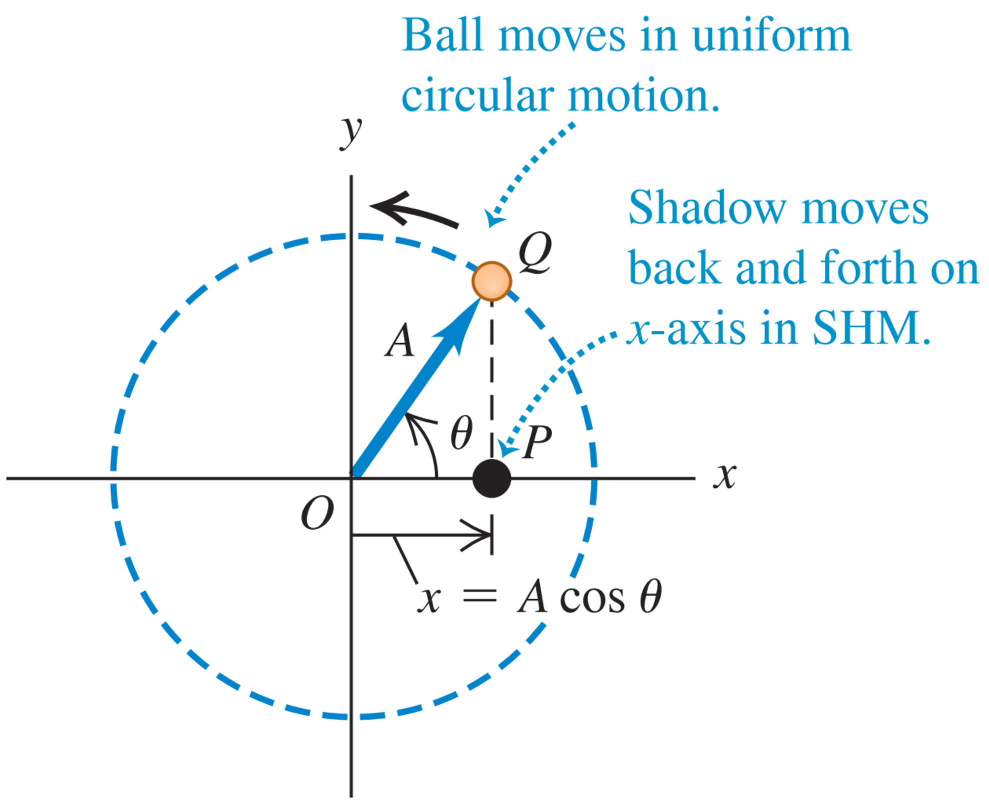
$$\omega = \frac{2\pi}{T} = 2\pi f \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

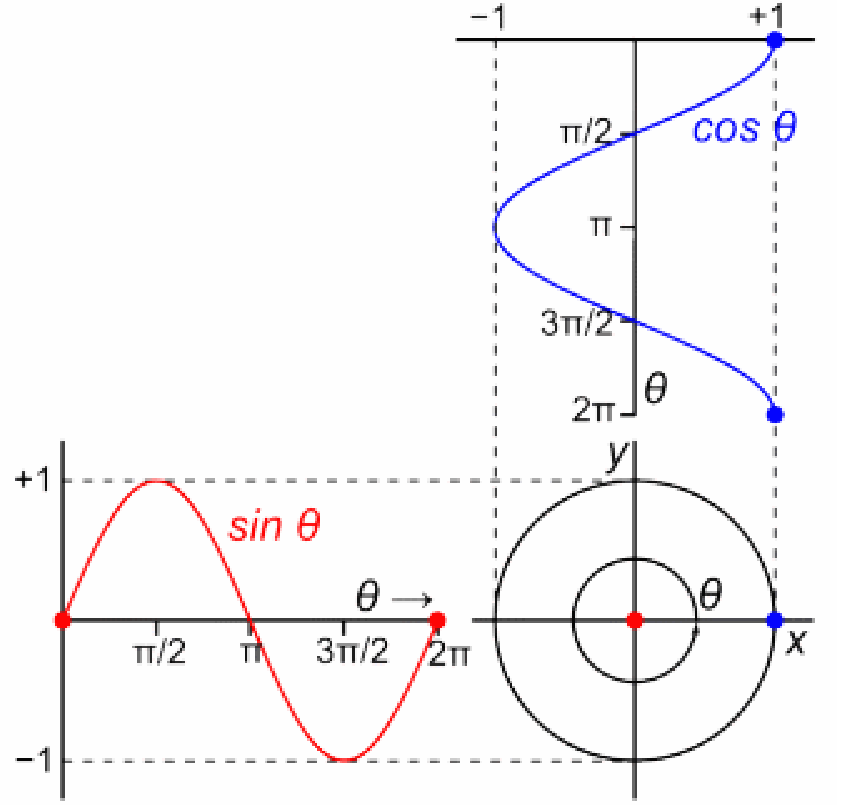
简谐运动方程也可以利用其和**匀速圆周运动**的关系来描述：

一个匀速圆周运动的物体在其直径上的投影 *x随时间的变化*，即为简谐运动。

因此有：$x = A\cos\theta$　其中，θ为任一时刻 t 物体的角位置 $\theta = \omega t + \varphi$，$\varphi$ 为起始角位置。

则立刻可得：$x = A\cos(\omega t + \varphi)$

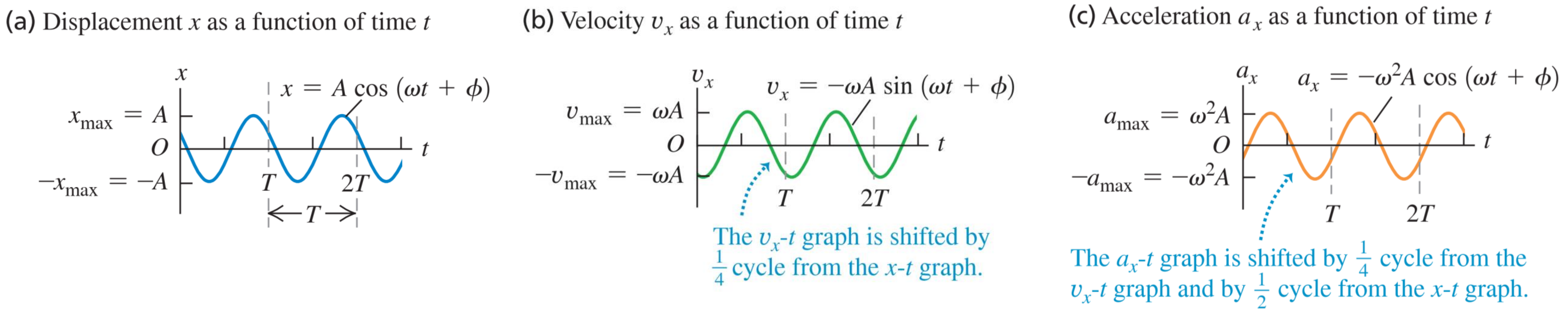




简谐运动的**速度方程、加速度方程：**

$$v_x = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \qquad \text{(velocity in SHM)}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi) \qquad \text{(acceleration in SHM)}$$



C. 简谐振动的能量

动能： $K = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega^2A^2\mathrm{Sin}^2(\omega t+\varphi)$

势能： $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\mathrm{Cos}^2(\omega t+\varphi) = \frac{1}{2}m\omega^2A^2\mathrm{Cos}^2(\omega t+\varphi)$

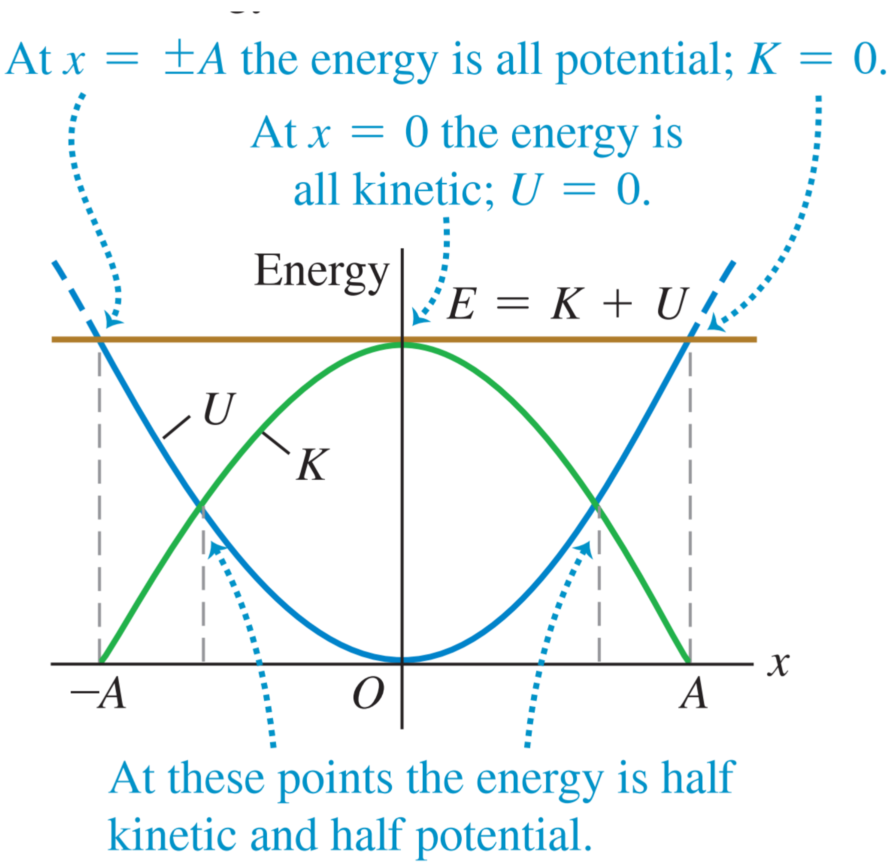
$$\omega^2 = \frac{k}{m} \qquad \cos^2\alpha + \sin^2\alpha = 1.$$

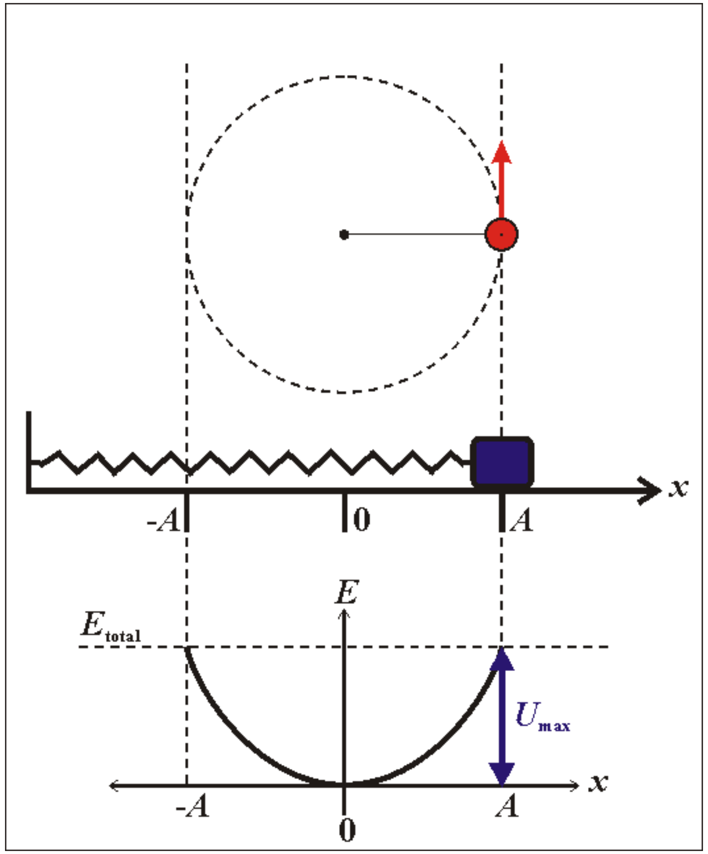
所以总的**机械能**：

Total mechanical energy in simple harmonic motion → $E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \text{constant}$

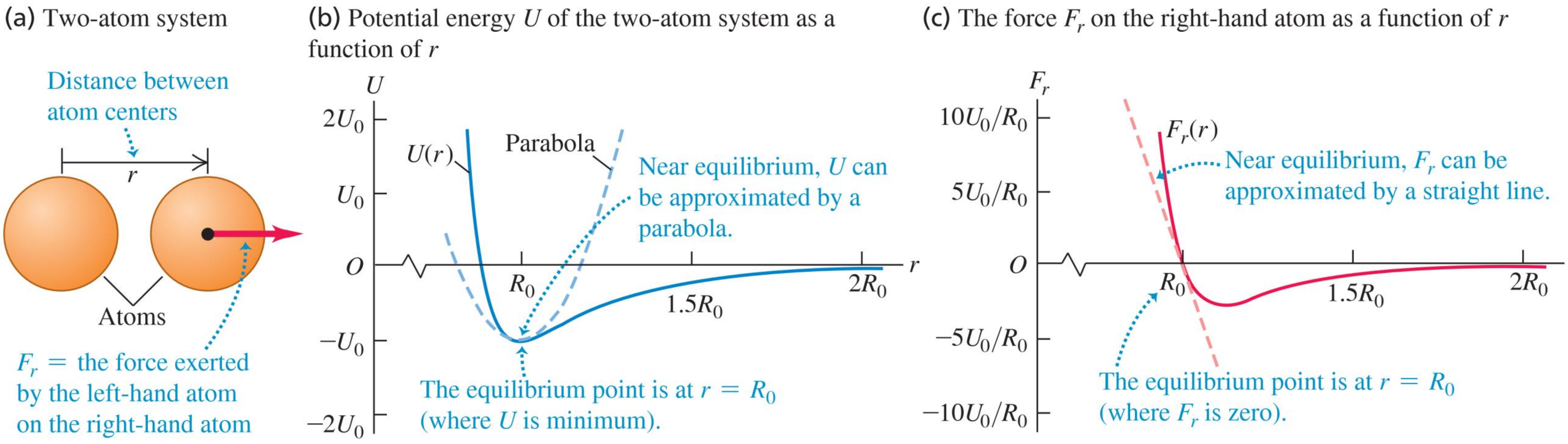
Mass (m); Force constant of restoring force (k); Velocity (v_x); Displacement (x); Amplitude (A)

简谐振动的总机械能守恒 $E=\frac{1}{2}kA^2$，只和最大振幅A有关





分子振动（molecular vibration）



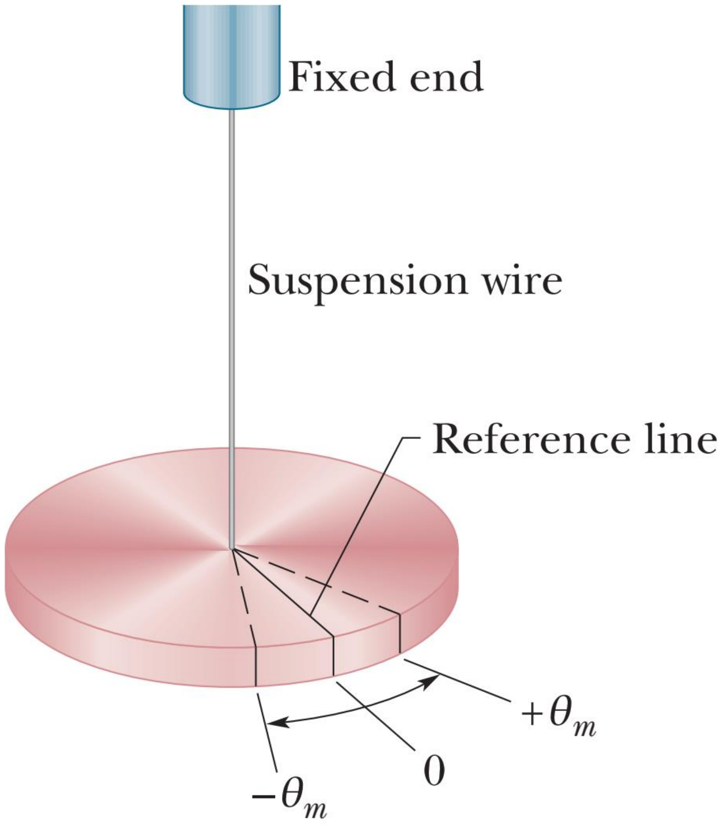
$$U = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right]$$

$$F_r = -\frac{dU}{dr} = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right]$$

小振幅时，近似满足胡克定律： $F_r \approx -\left(\frac{72U_0}{R_0^2}\right)x$

D. 扭摆（torsion pendulum）

扭摆是一种角简谐振动（angular simple harmonic motion）



扭矩： $\tau = -\kappa\theta.$

$$\theta = \Theta\cos(\omega t + \phi)$$

κ 为扭转常数　　Θ 为最大角位移（角幅度）

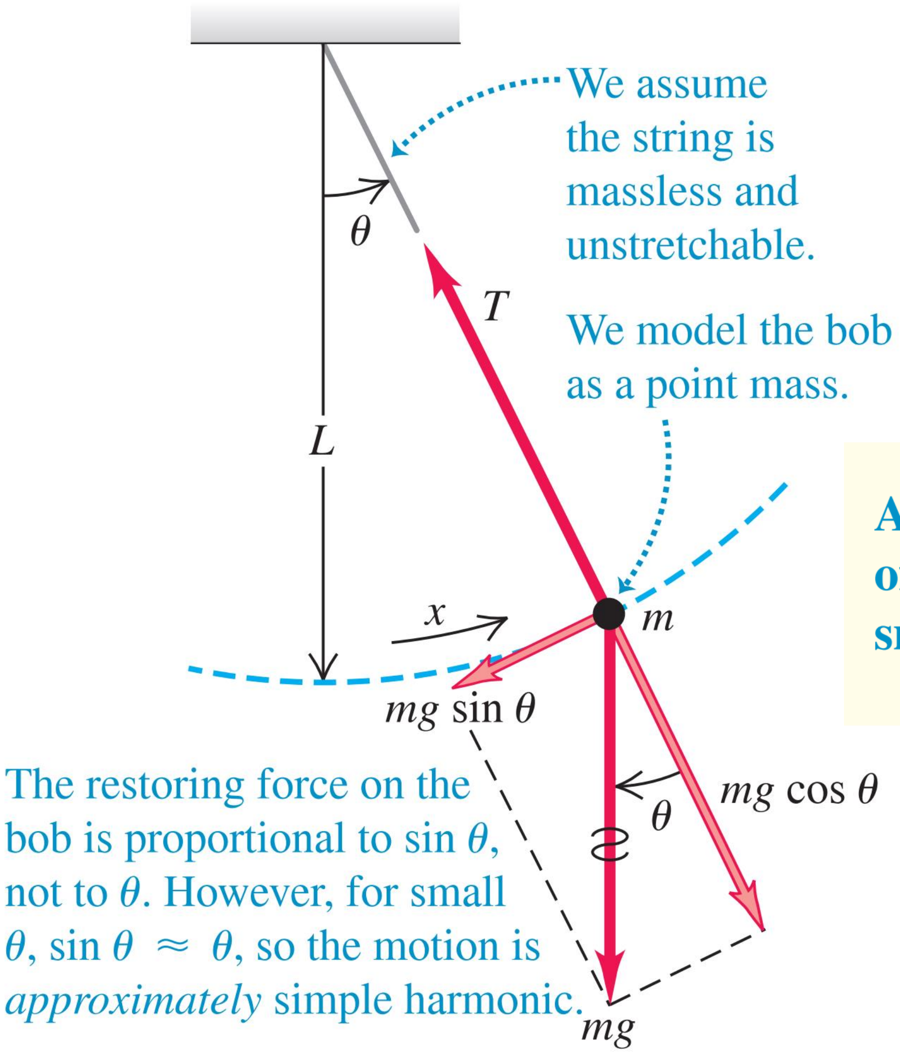
Angular simple harmonic motion → $\omega = \sqrt{\frac{\kappa}{I}}$ (Angular frequency) and $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ (Frequency)

Torsion constant divided by moment of inertia

I 为圆盘的转动惯量

E. 单摆（simple pendulum）

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots \approx \theta$$



$$F_\theta = -mg\sin\theta$$

振幅很小时，$\mathrm{Sin}\theta \approx \theta$，近似为简谐运动：

$$F_\theta = -mg\theta = -mg\frac{x}{L} = -\frac{mg}{L}x$$

角频率为：

Angular frequency of simple pendulum, small amplitude

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$$

Acceleration due to gravity (g); Pendulum length (L); Pendulum mass (cancels) (m)

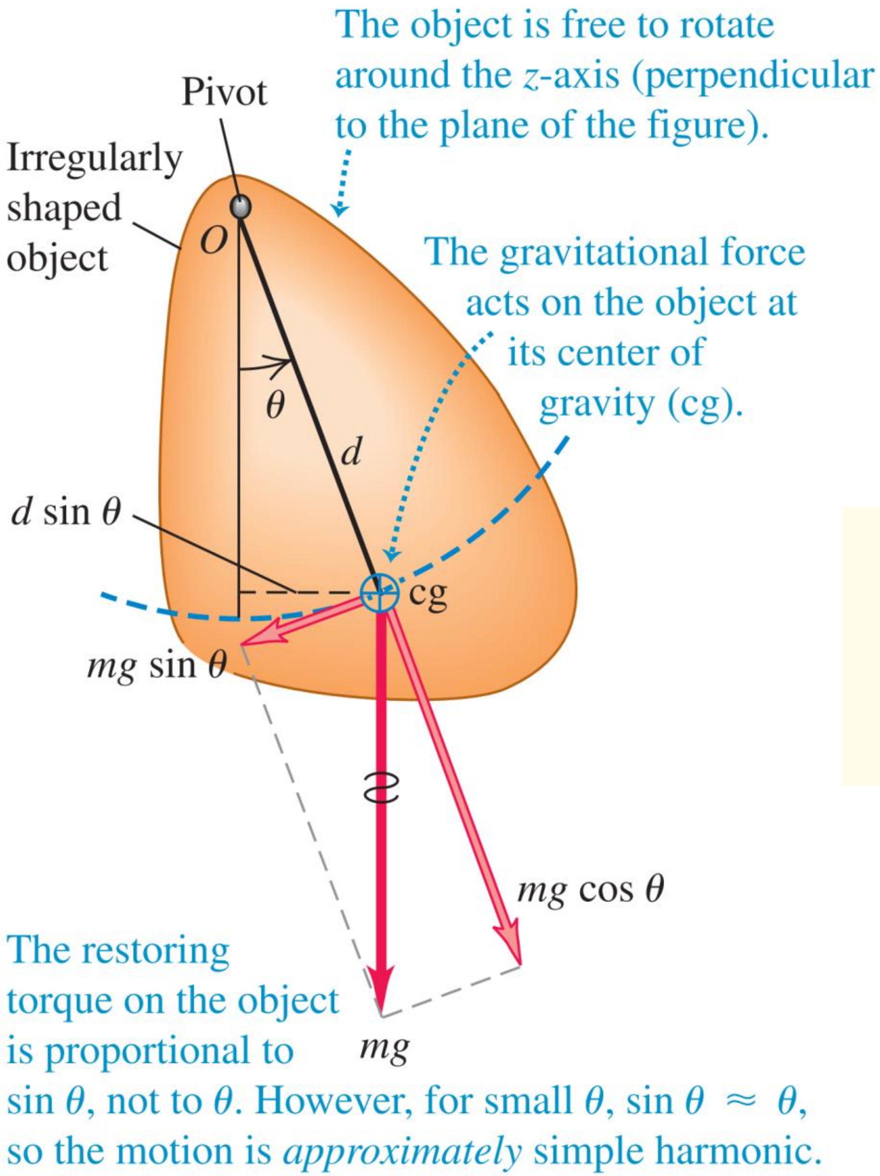
周期为：

Period of simple pendulum, small amplitude

$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

Angular frequency (ω); Frequency (f); Pendulum length (L); Acceleration due to gravity (g)

F. 复摆（physical pendulum）

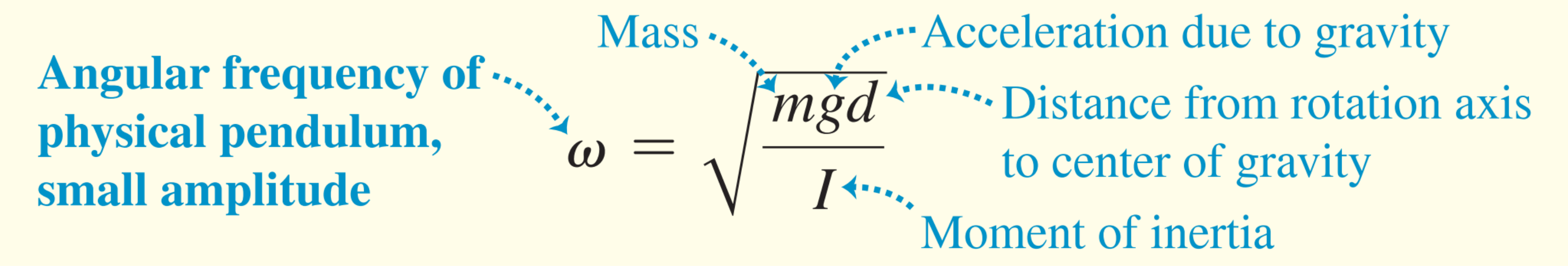


$$\tau_z = -(mg)(d\sin\theta)$$

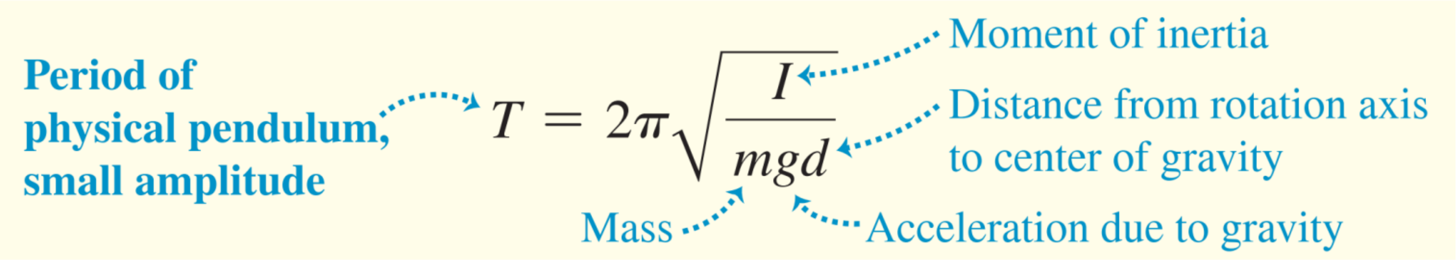
振幅很小时，Sinθ ~ θ，近似为简谐运动：

$$\tau_z = -(mgd)\theta$$

角频率为：

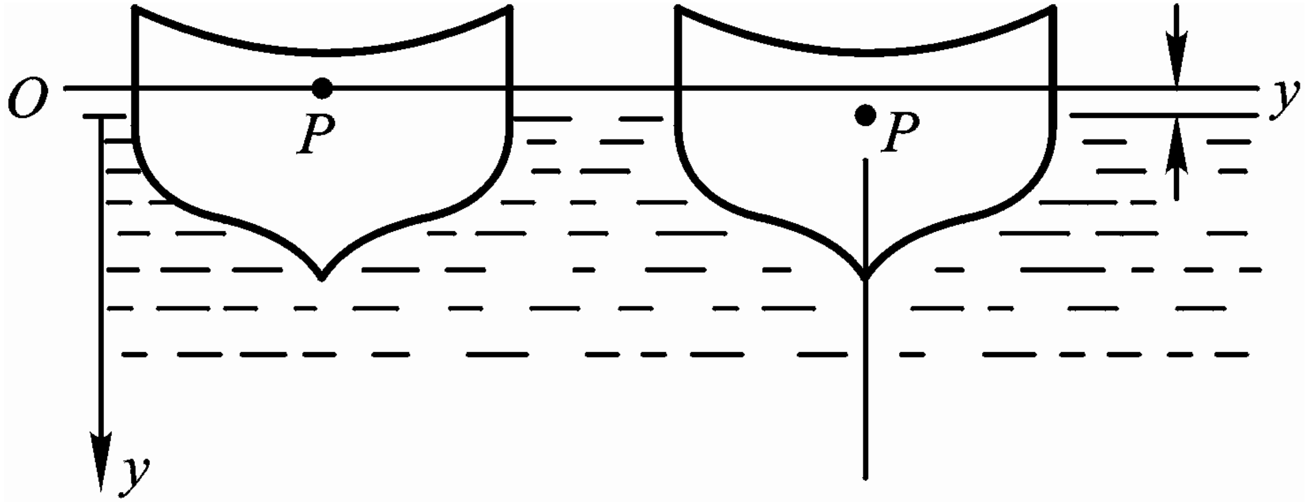


周期为：



例13-1 一质量为m 的平底船，其平均水平截面积为S，吃水深度为h，如不计水的阻力，求此船在竖直方向的振动周期。

解： 船静止时浮力与重力平衡，



船在任一位置时，以水面为坐标原点，竖直向下的坐标轴为y 轴，船的位移用y 表示。

船的位移为y时船所受合力为

$$F=-(h+y)\rho Sg+mg=-y\rho Sg$$

$$F=-\omega^2 y m$$

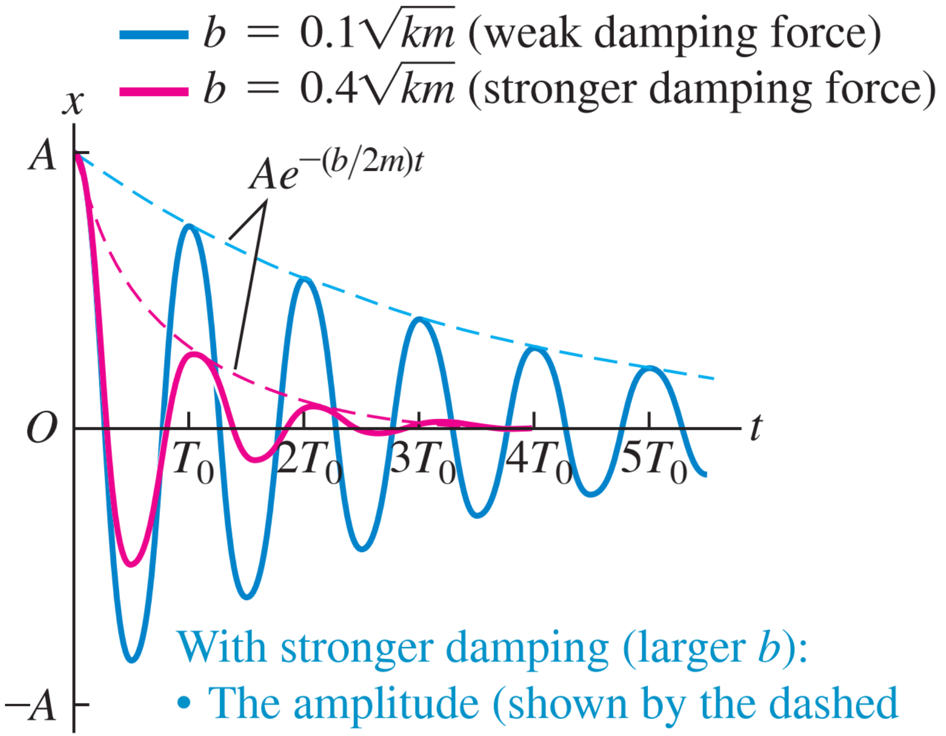
⇨ 船在竖直方向做简谐振动，其角频率和周期为

$$\omega=\sqrt{\frac{\rho Sg}{m}}\quad,\qquad T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{\rho gS}}$$

$$m=\rho Sh \quad \Rightarrow \quad T=2\pi\sqrt{\frac{h}{g}}$$

G. 阻尼振动（damped oscillation）

The decrease in amplitude caused by dissipative forces is called **damping**



With stronger damping (larger b):

- The amplitude (shown by the dashed curves) decreases more rapidly.
- The period T increases (T_0 = period with zero damping).

考虑摩擦力 $-bv_x$，b 为阻尼系数

则合力为： $\sum F_x = -kx - bv_x$

$$-kx - bv_x = ma_x \qquad \text{or} \qquad -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

阻尼振动方程：

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$$

- **Displacement of oscillator, little damping**: x
- Initial amplitude: A
- Damping constant: b
- Mass: m
- Time: t
- Angular frequency of damped oscillations: ω'
- Phase angle: ϕ

Angular frequency of oscillator, little damping

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

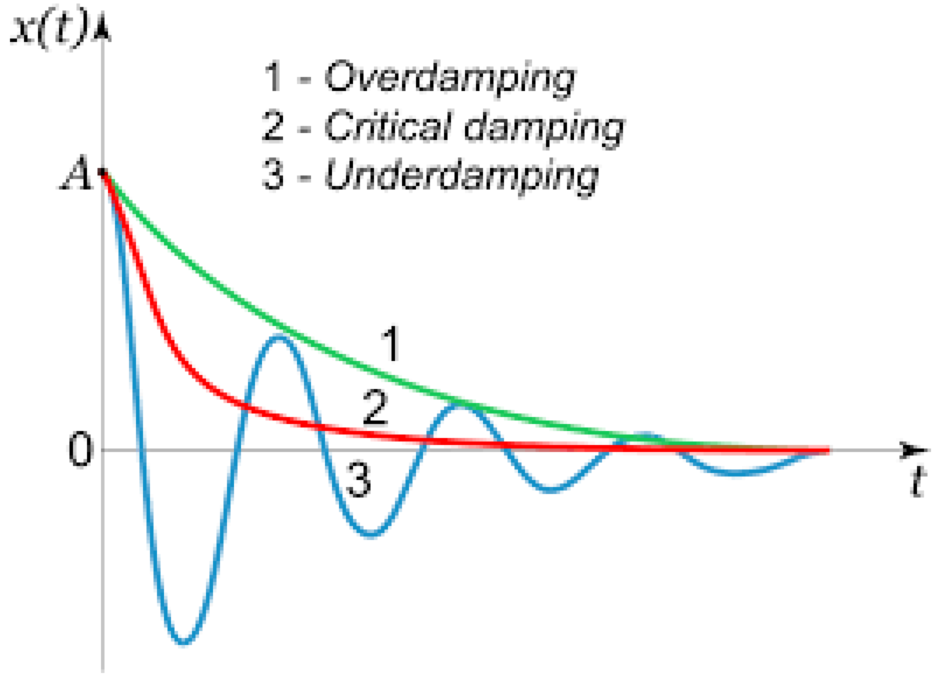
k: Force constant of restoring force; b: Damping constant; m: Mass

$$b = 2\sqrt{km}$$

临界阻尼（critical damping），刚好不发生振荡

>此值，过阻尼（overdamping），不发生振荡

<此值，欠阻尼（underdamping），衰减振荡



Q值（品质因子，quality factor）：

由于阻尼的存在，物体在每完成一个周期的振荡，其总能量 E 就衰减去$(\Delta E)_T$

定义品质因子Q，用来度量阻尼振动中保持能量的能力：

$$Q = 2\pi \frac{E}{(\Delta E)_T}$$

Q值越大，则阻尼和能量损耗越小。

Q值可能随时间改变，但大部分的阻尼振动中，Q值近似与时间无关，是个常数。

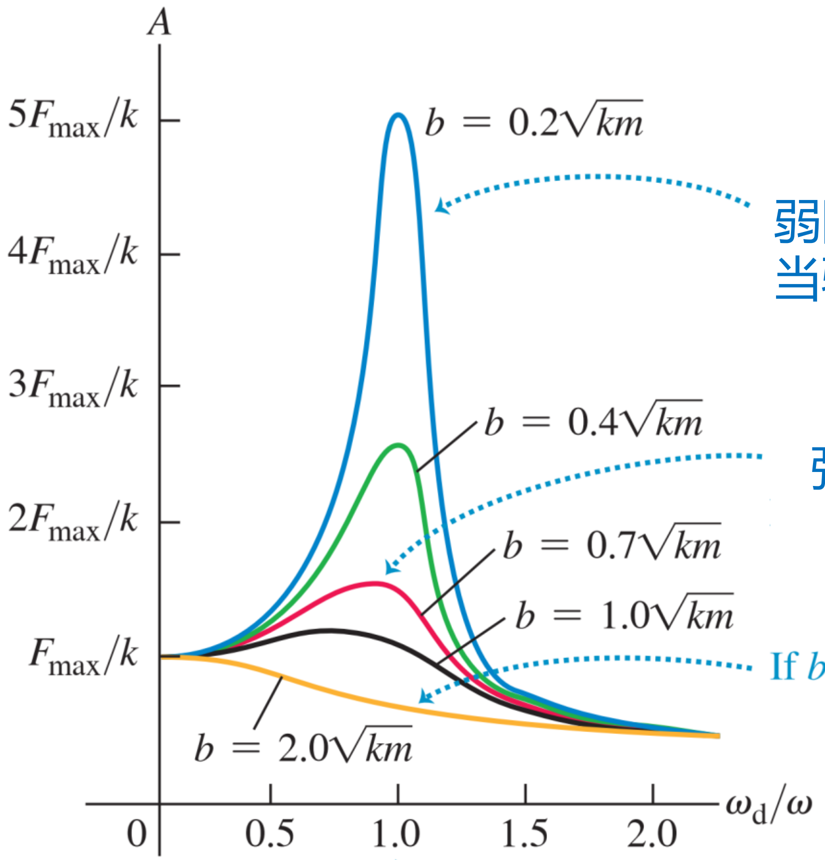
H. 受迫振动（forced oscillation）

系统在周期性驱动力作用下的振动，
振动角频率等于驱动力的角频率ω_d

Amplitude of a forced oscillator:

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

Maximum value of driving force (F_{max}); Damping constant (b); Force constant of restoring force (k); Mass (m); Driving angular frequency (ω_d)



弱阻尼振动表现出尖锐的共振峰，
当驱动频率ω_d接近本征频率ω时，
振幅最大

强的阻尼导致峰值的降低和峰的
展宽并向低频移动

If $b \geq \sqrt{2km}$, the peak disappears completely.

Driving frequency ω_d equals natural angular frequency ω of an undamped oscillator.

共振：

$$\omega_d = \omega \quad \text{(resonance)},$$

此时振幅*A*将达很大的值，这种现象称为共振

建筑的最大危害：

共振！！！

专家组通过对风致振动与结构累积损伤的重点分析，认为桅杆风致涡激共振和大厦及桅杆动力特性改变的耦合，造成了赛格大厦的有感振动。大厦使用20余年后，局部楼层压型钢板组合楼板及桅杆连接点等累积损伤，使结构频率、阻尼比等动力特性发生了改变，桅杆和大厦主体结构具有了共同振动频率，形成了共振的必要条件。

I. 简谐振动的合成

(a)：同方向、同频率

两谐振动

$$x_1 = A_1 \cos(\omega_0 t + \alpha_1)$$

$$x_2 = A_2 \cos(\omega_0 t + \alpha_2)$$

合位移

$$x = x_1 + x_2$$

$$= A_1 \cos(\omega_0 t + \alpha_1) + A_2 \cos(\omega_0 t + \alpha_2)$$

$$= A\cos(\omega_0 t + \alpha)$$

结论：同方向、同频率两简谐振动的合成，合运动仍是同频率的简谐振动.

其中

$$A = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1A_2\cos(\alpha_2 - \alpha_1)}$$

$$\tan\alpha = \frac{A_1\sin\alpha_1 + A_2\sin\alpha_2}{A_1\cos\alpha_1 + A_2\cos\alpha_2}$$

(b)：同方向、不同频率

1. 合成

两谐振动

$$x_1 = A_1 \cos(\omega_{10} t + \alpha_1)$$

$$x_2 = A_2 \cos(\omega_{20} t + \alpha_2)$$

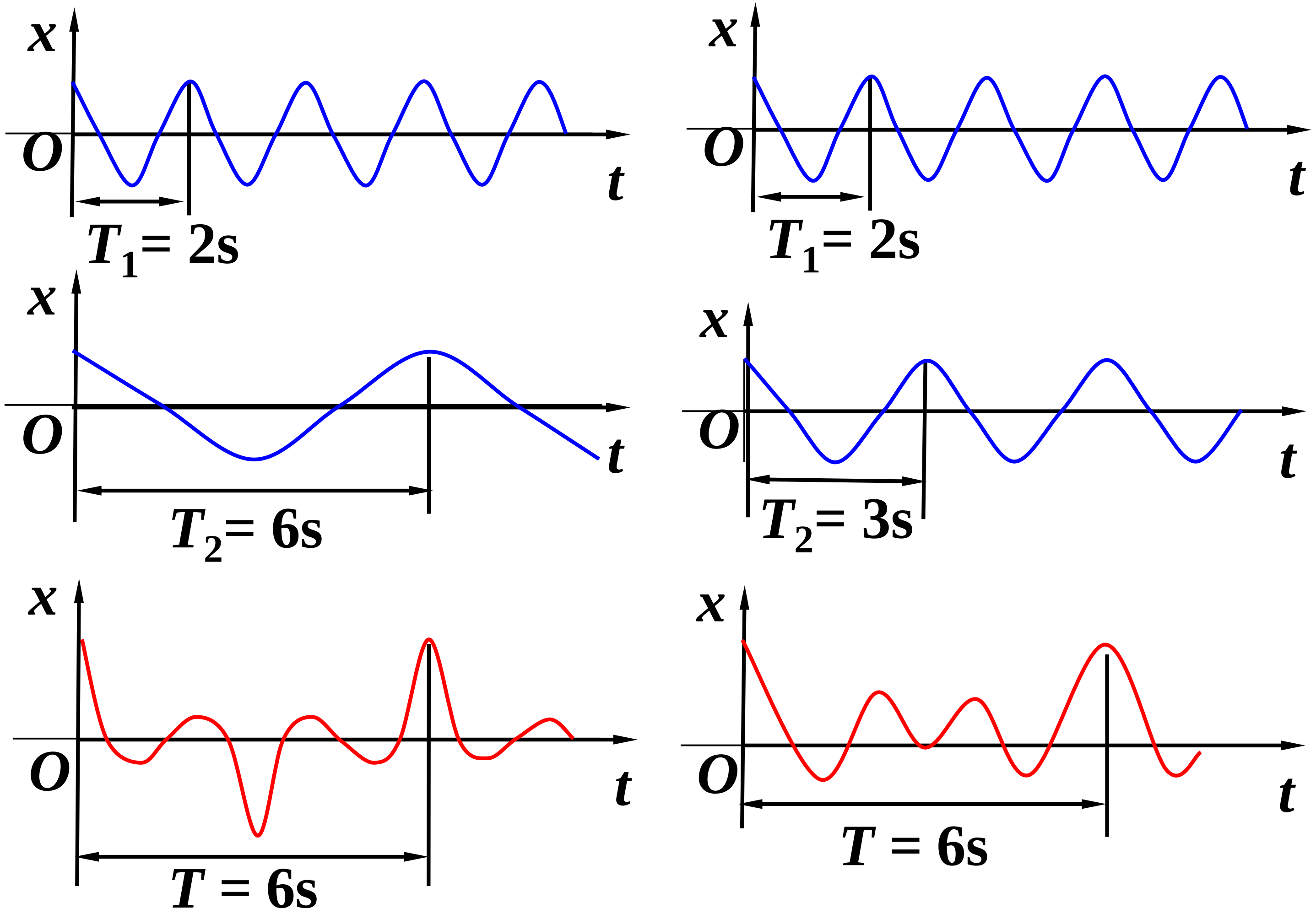
设 $A_1 = A_2 = A,\ \alpha_1 = \alpha_2 = 0$

$$x_1 = A\cos\omega_{10} t$$

$$x_2 = A\cos\omega_{20} t$$

合位移 $x = x_1 + x_2 = A\cos\omega_{10} t + A\cos\omega_{20} t$

可用两分振动的位移时间曲线得出合振动的位移时间曲线，合振动不再是简谐振动，但却有周期性.


x
O
t
T_1= 2s
x
O
t
T_1= 2s
x
O
t
T_2= 6s
x
O
t
T_2= 3s
x
O
t
T = 6s
x
O
t
T = 6s

合振动周期称主周期，主周期有两个特点:

(1)主周期是分振动周期的整数倍,

(2)主周期是分振动周期的最小公倍数.

2. 拍

合振动 $x = x_1 + x_2 = A\cos\omega_{10}t + A\cos\omega_{20}t$

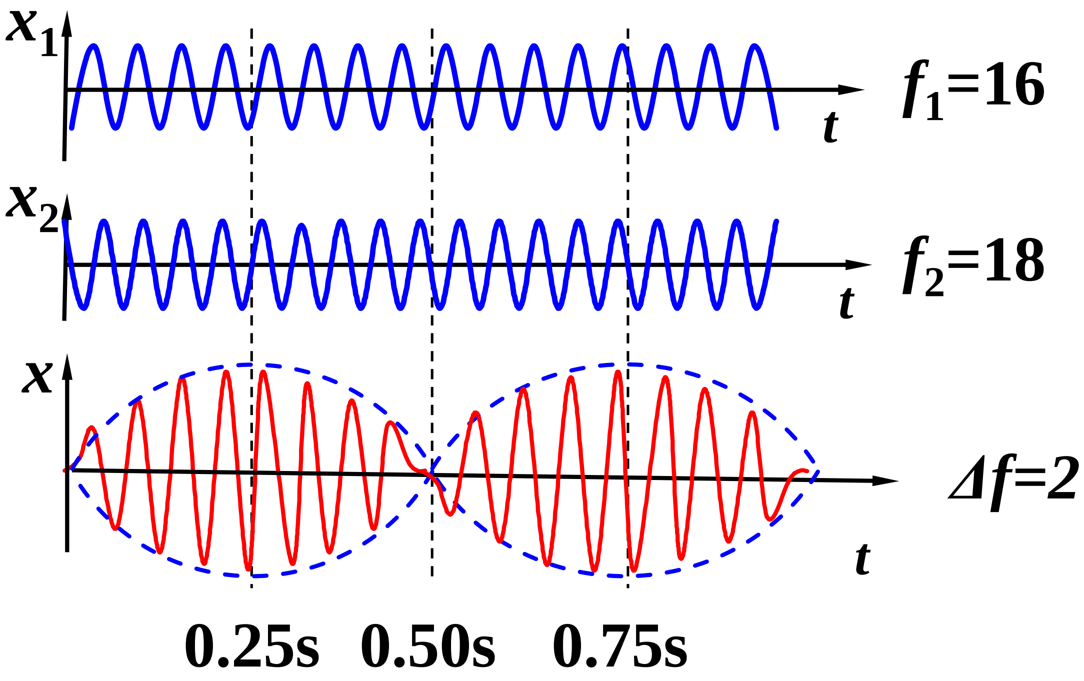
$$= 2A\cos\frac{\omega_{20}-\omega_{10}}{2}t\cos\frac{\omega_{20}+\omega_{10}}{2}t$$

设 $\omega_{10}+\omega_{20} >> |\omega_{10}-\omega_{20}|$ 频率非常接近

$\cos\frac{\omega_{20}-\omega_{10}}{2}t$ 随 t 变化慢

——准简谐振动.

$\cos\frac{\omega_{20}+\omega_{10}}{2}t$ 随t变化快



●合成的振动相当于振幅随时间缓慢变化的简谐振动。

●振动频率与原两振动频率几乎相等。

拍——频率较大但相差不大的两个同方向简谐振动合成时产生合振动振幅周期性变化的现象.

拍频——单位时间内振动加强或减弱的次数. $\omega_{拍}=|\omega_{20}-\omega_{10}|$ $f_{拍}=|f_2-f_1|$

（c）：垂直方向，李萨如图形

1. 频率相同

两谐振动 $x = A_1\cos(\omega_0 t + \alpha_1)$

$y = A_2\cos(\omega_0 t + \alpha_2)$

消去 t

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1 A_2}\cos(\alpha_2 - \alpha_1) = \sin^2(\alpha_2 - \alpha_1)$$

合成轨迹一般为一椭圆, 两振幅相等时为圆. 具体的来说：形状由相差决定.

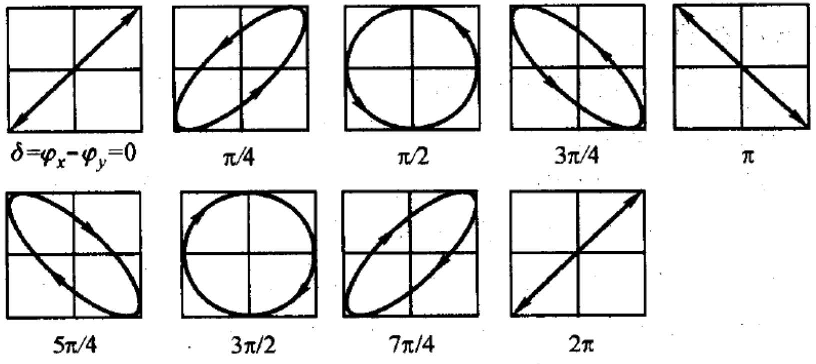
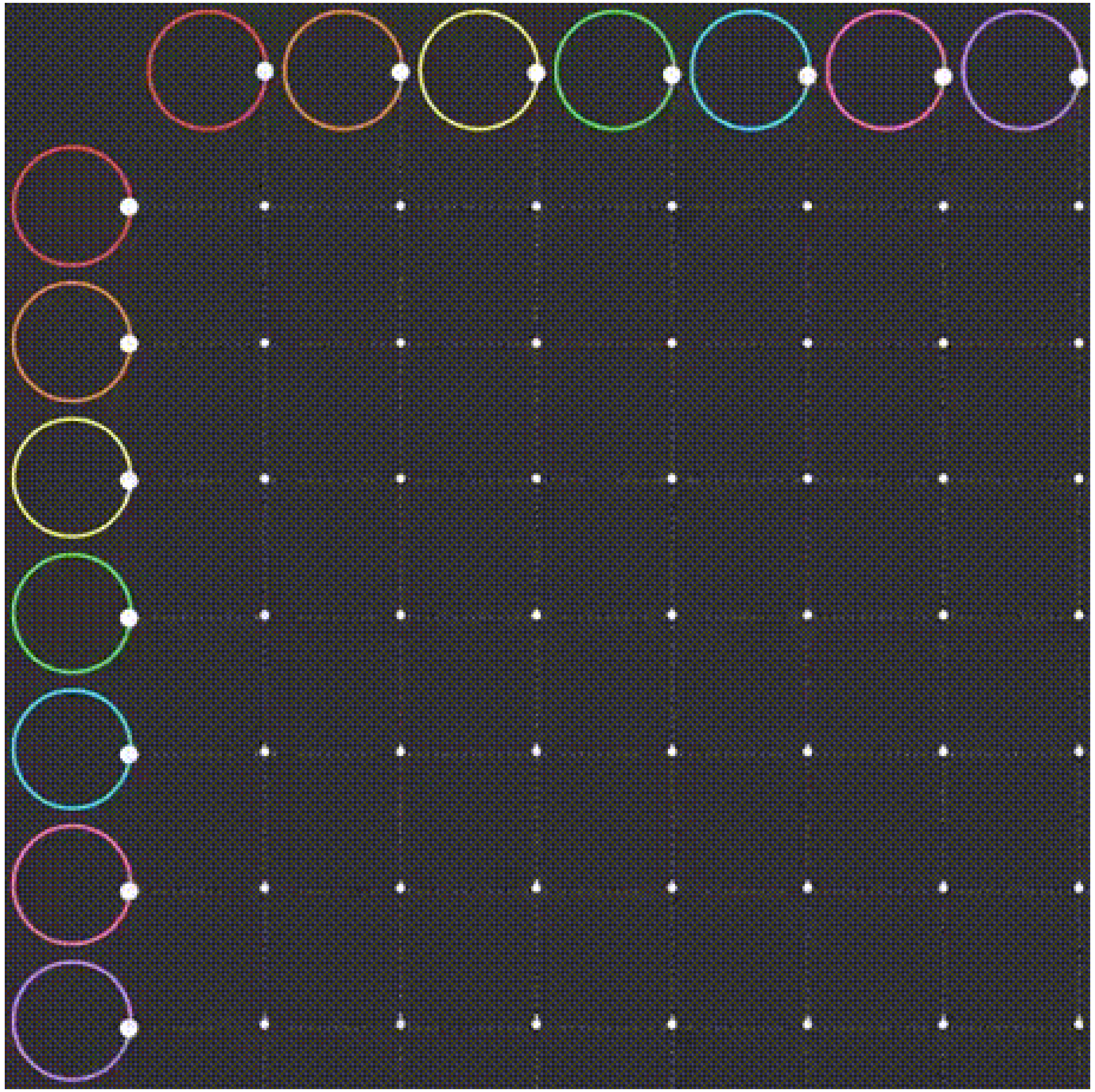


图 8.2－6　不同相位差对应的椭圆

说明：任何一个直线简谐振动，椭圆运动或匀速圆周运动都可分解为两个相互垂直的简谐振动.

2. 频率不同，为整数比——李萨如图形



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随时间的变化