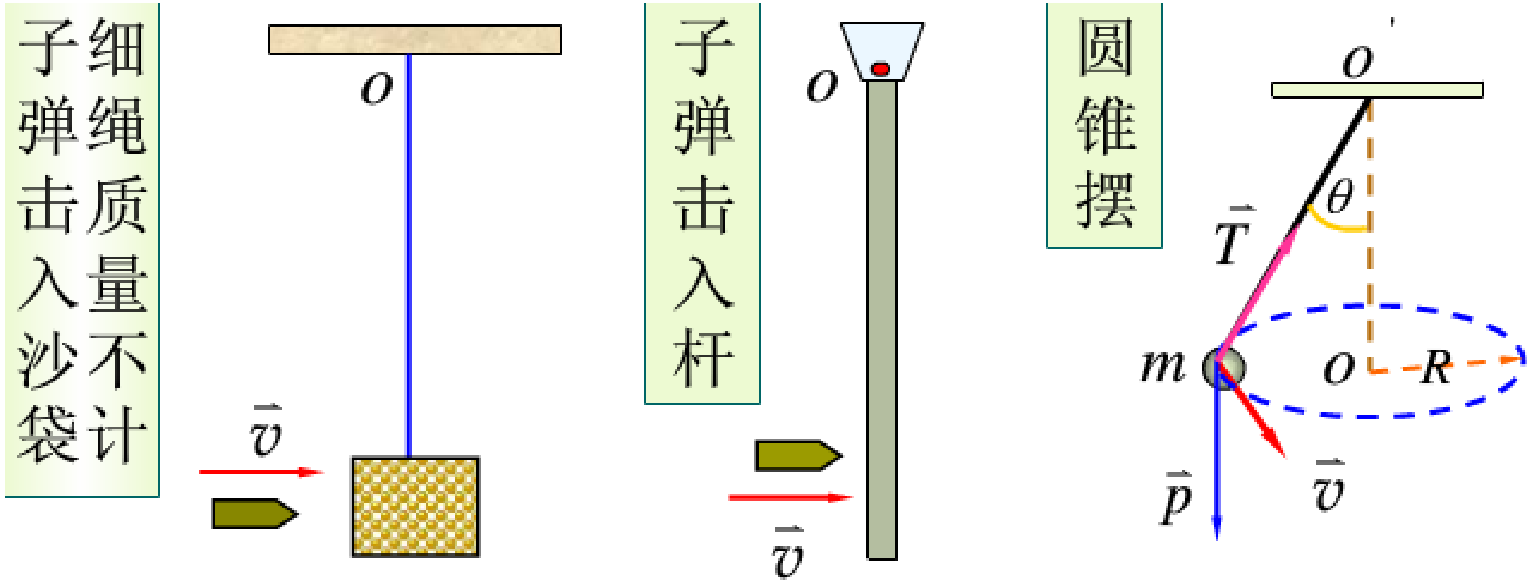
角动量守恒也是一个独立的规律，即它并不包含在能量守恒和动量守恒规律中。

例：系统的动量、角动量、动能是否守恒



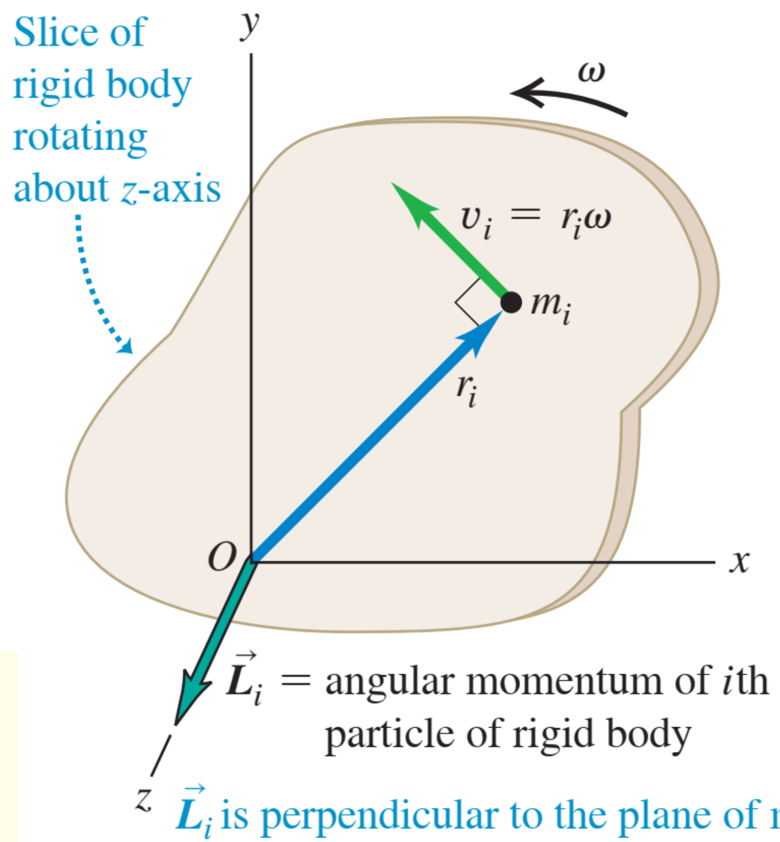
	以子弹和沙袋为系统	以子弹和杆为系统	圆锥摆系统
动量:			
角动量:			
机械能:			

注意：前两个系统特指碰撞前后，但还没有发生转动

10-11 刚体的角动量

$$L_i = m_i(r_i\omega)\,r_i = m_i r_i^{\,2}\omega$$

$$L = \sum L_i = \left(\sum m_i r_i^{\,2}\right)\omega = I\omega$$



$\vec{L}_i$ is perpendicular to the plane of motion (if the origin O is in that plane) and has magnitude $L_i = m_i v_i r_i = m_i r_i^{\,2}\omega$.

Angular momentum of a rigid body rotating around a symmetry axis

$$\vec{L} = I\vec{\omega}$$

Moment of inertia of rigid body about symmetry axis

Angular velocity vector of rigid body

For a system of particles:

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

Sum of external torques on the system

Rate of change of total angular momentum $\vec{L}$ of system

刚体的角动量定理: 质点在$t1 \rightarrow t2$时间内所受合外力矩的冲量矩等于该段时间内刚体角动量的增量

10.12 刚体的角动量守恒定律

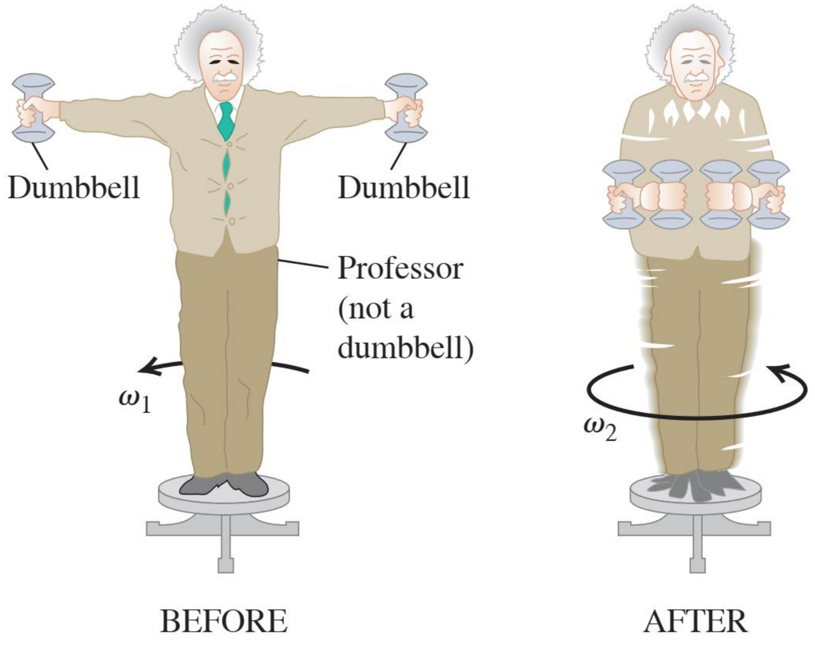
$$\sum\vec{\tau} = d\vec{L}/dt. \text{ If } \sum\vec{\tau} = \mathbf{0}, \text{ then } d\vec{L}/dt = \mathbf{0}, \text{ and } \vec{L} \text{ is constant.}$$

CONSERVATION OF ANGULAR MOMENTUM **When the net external torque acting on a system is zero, the total angular momentum of the system is constant (conserved).**

若体系所受的合外力矩为零, 则体系的总角动量保持不变

- 角动量定理适用于一切转动问题,大至天体,小至粒子、电子...
- 角动量守恒定律和能量守恒、动量守恒一样是普适的守恒律，适用于从星系到微观粒子的一切物体

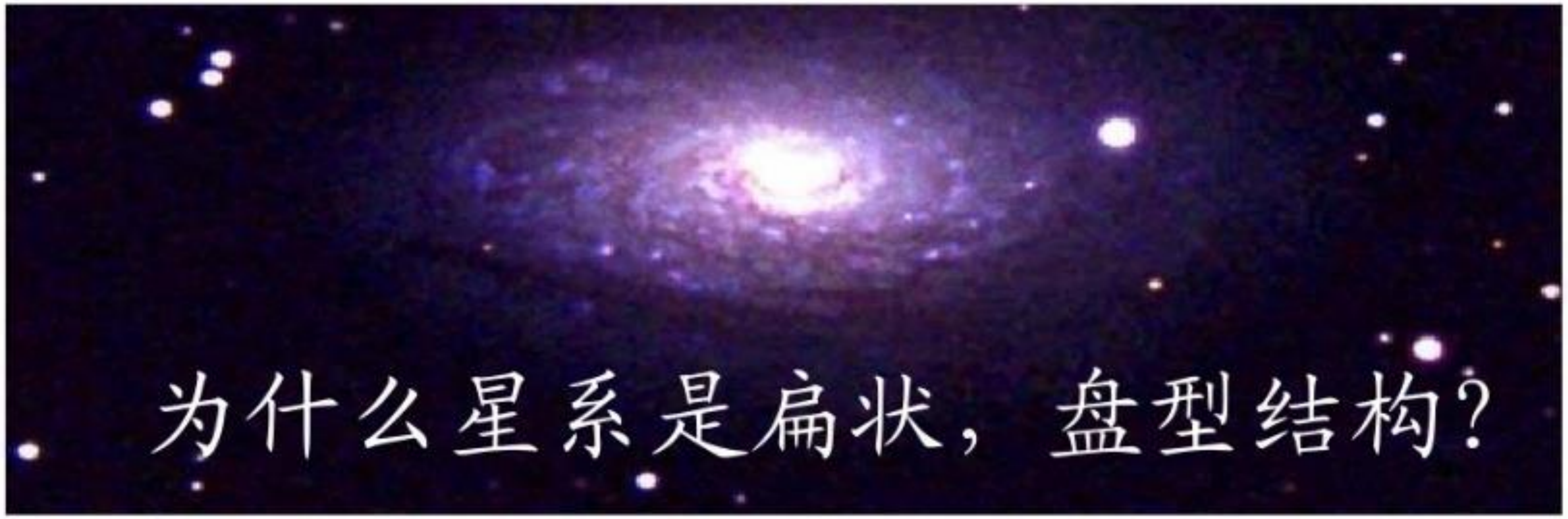
角动量守恒的实验



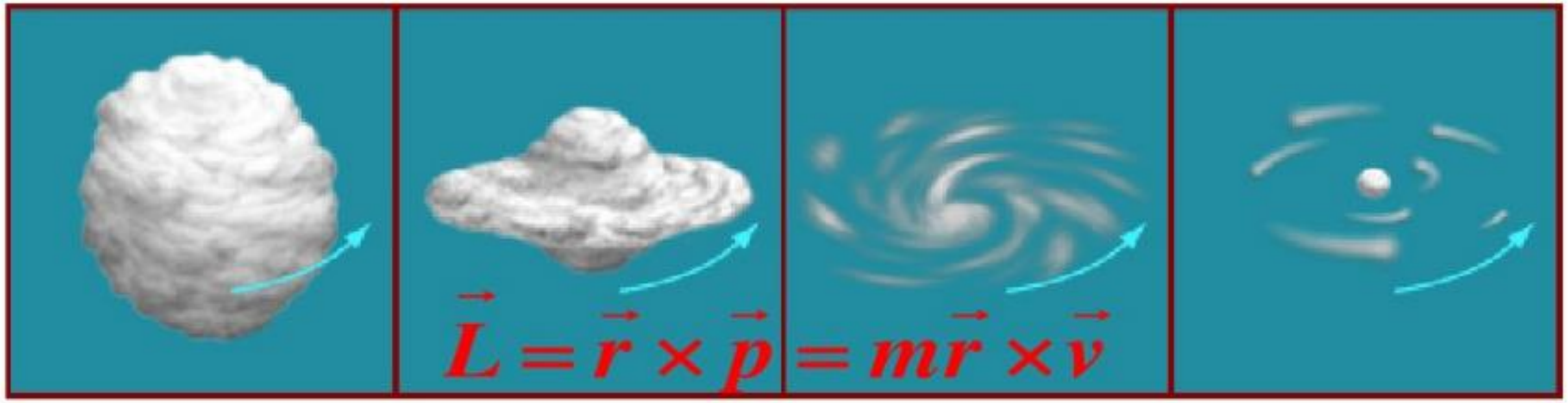
角动量守恒的分量

角动量守恒也是一个矢量关系，它包括三个不变的量，若合外力矩在x、y、z三个方向的分量分别为零，则分别有

$$L_x = \text{不变量}, \quad L_y = \text{不变量}, \quad L_z = \text{不变量}$$

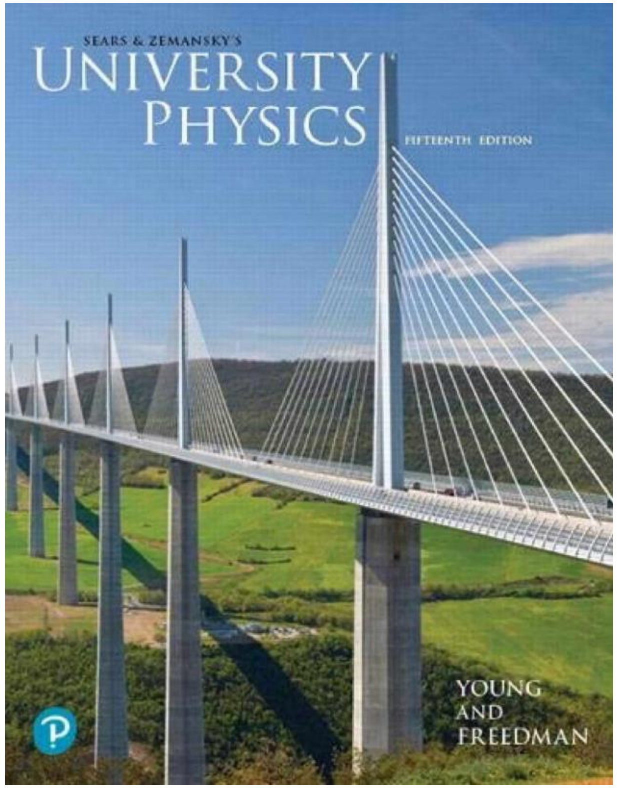


18世纪哲学家提出星云说，认为太阳系是由气云组成的。气云原来很大，由自身引力而收缩，最后聚集成一个个行星、卫星及太阳本身。但是万有引力为什么不能把所有的天体吸引在一起而是形成一个扁平的盘状？



康德认为除了引力还有斥力，把向心加速的天体散射到一个方向。19世纪数学家拉普拉斯完善了康德的星云说，***指出旋转盘状结构的成因是角动量守恒。***

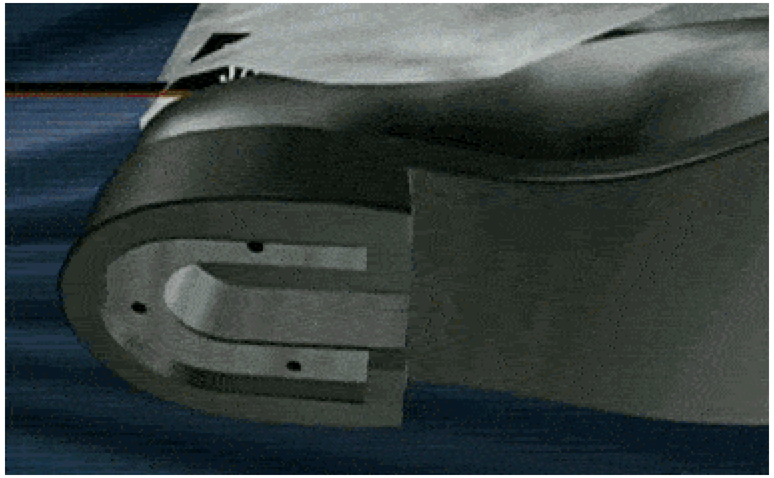
平衡
Equilibrium



Examples

世界最高的三塔连体建筑


MJackson HD



A. 平衡条件

什么是平衡?

- 没有加速度

直线加速度=0 & 转动加速度=0

力为零 ｜ 力矩为零

平衡的两个条件

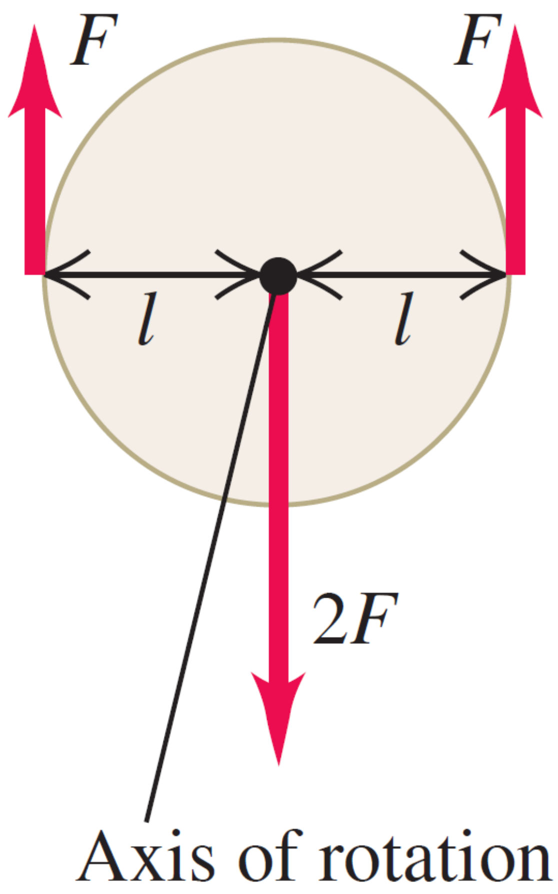
静止物体的质心必须静止　　　　净合外力必须为零

First condition for equilibrium: For the center of mass of an object at rest to remain at rest … $\sum \vec{F} = 0$ ← … the *net external force* on the object must be *zero*.

非旋转物体必须保持非旋转　　　　净合外力矩必须为零

Second condition for equilibrium: For a nonrotating object to remain nonrotating … $\sum \vec{\tau} = 0$ ← …the *net external torque around any point* on the object must be *zero*.

Example 1

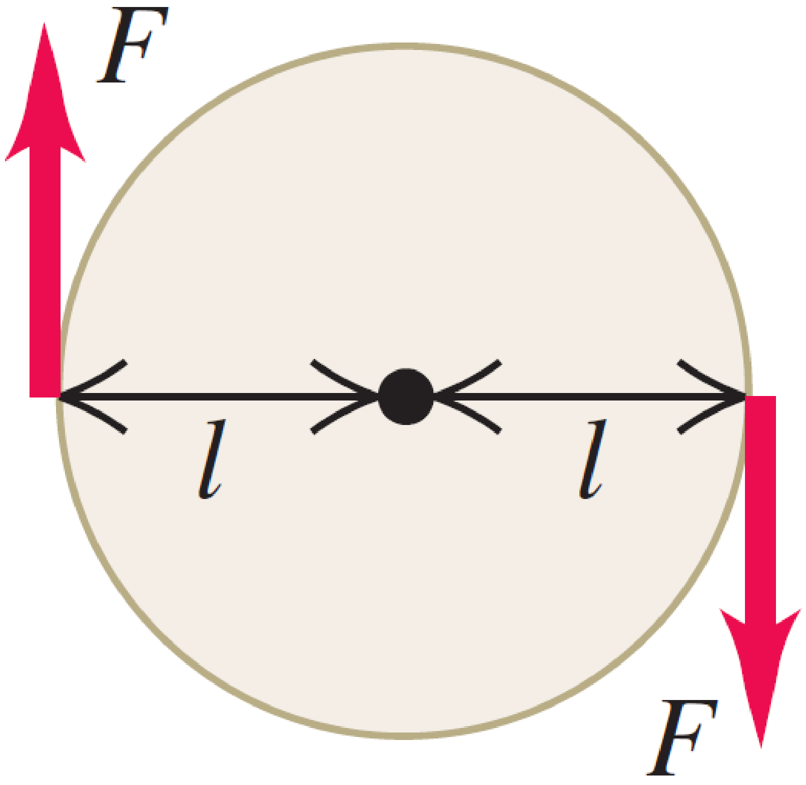


First condition satisfied:
Net force = 0, so object at rest has no tendency to start moving as a whole.

Second condition satisfied:
Net torque about the axis = 0, so object at rest has no tendency to start rotating.

√ **In static equilibrium (静态平衡)**

Example 2



First condition satisfied: Net force = 0, so object at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so object at rest will start rotating clockwise.

x Not in equilibrium (非平衡)

Real examples of dynamic equilibrium

非洲 自行车杂技

清华 天机芯 无人驾驶

华为 稚晖君

请大家思考：能否造出自平衡的独轮车？

质心和重心

center of mass and center of gravity

质心定义：

Position vector of center of mass of a system of particles

Position vectors of individual particles

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

Masses of individual particles

什么时候质心和重心重合?

---重力加速度在整个质心系中不变时，二者重合。

质点系所受重力力矩

单个质点所受重力力矩：

$$\vec{\tau}_i = \vec{r}_i \times \vec{w}_i = \vec{r}_i \times m_i \vec{g}$$

所有质点所受重力力矩：

$$\begin{aligned}\vec{\tau} &= \sum_i \vec{\tau}_i = \vec{r}_1 \times m_1 \vec{g} + \vec{r}_2 \times m_2 \vec{g} + \cdots \\ &= (m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots) \times \vec{g} \\ &= \left(\sum_i m_i \vec{r}_i \right) \times \vec{g}\end{aligned}$$

质心所受重力力矩

所有质点所受重力力矩：

$$\vec{\tau} = \sum_i \vec{\tau}_i = \vec{r}_1 \times m_1\vec{g} + \vec{r}_2 \times m_2\vec{g} + \cdots$$
$$= (m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots) \times \vec{g}$$
$$= \left(\sum_i m_i\vec{r}_i\right) \times \vec{g}$$

质心所受重力力矩：

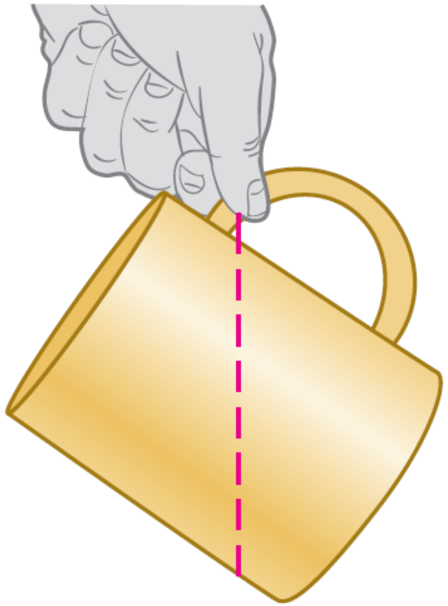
$$\vec{\tau} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots} \times M\vec{g} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \times M\vec{g}$$

$$\Rightarrow \boxed{\vec{\tau} = \vec{r}_{\mathrm{cm}} \times M\vec{g} = \vec{r}_{\mathrm{cm}} \times \vec{w}}$$

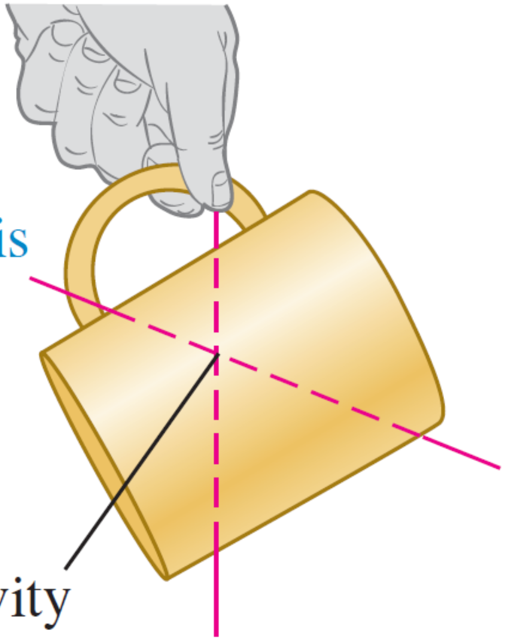
怎样找不规则物体的重心?

Where is the center of gravity of this mug?

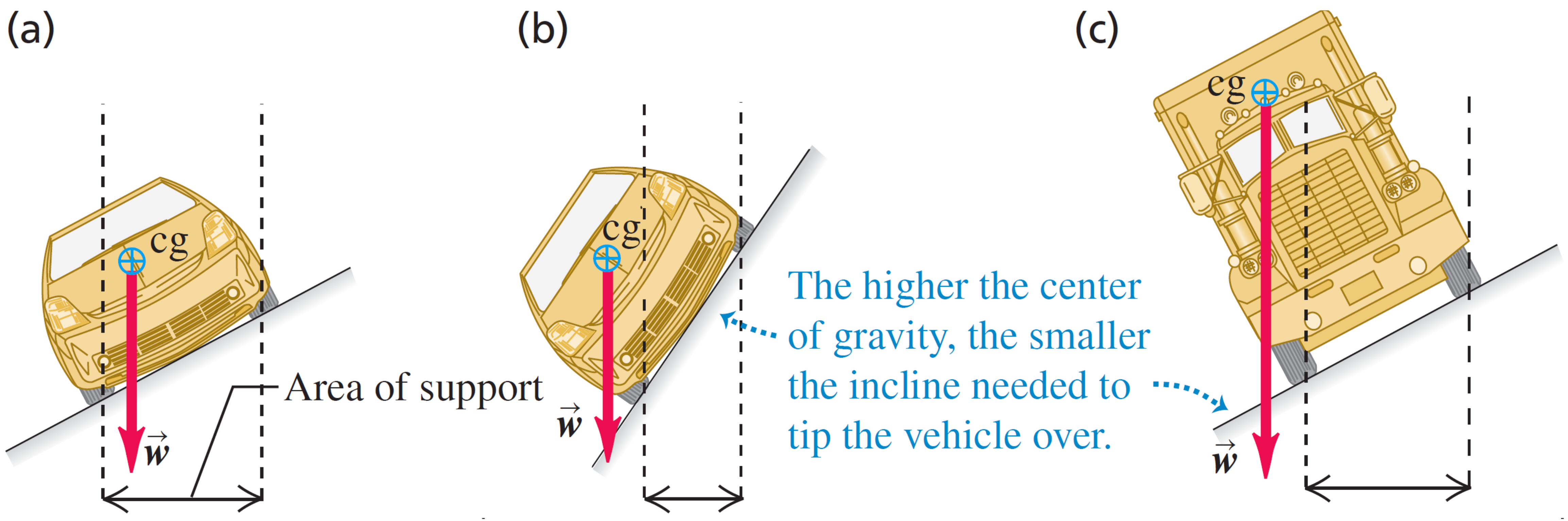
① Suspend the mug from any point. A vertical line extending down from the point of suspension passes through the center of gravity.



② Now suspend the mug from a different point. A vertical line extending down from this point intersects the first line at the center of gravity (which is inside the mug).



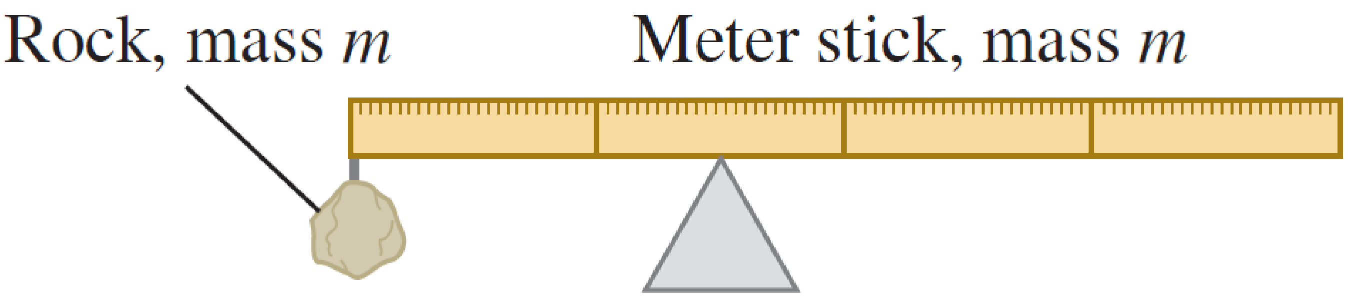
思考：下列那些情况不能保持平衡?



平衡　　　不平衡　　　不平衡

问题：

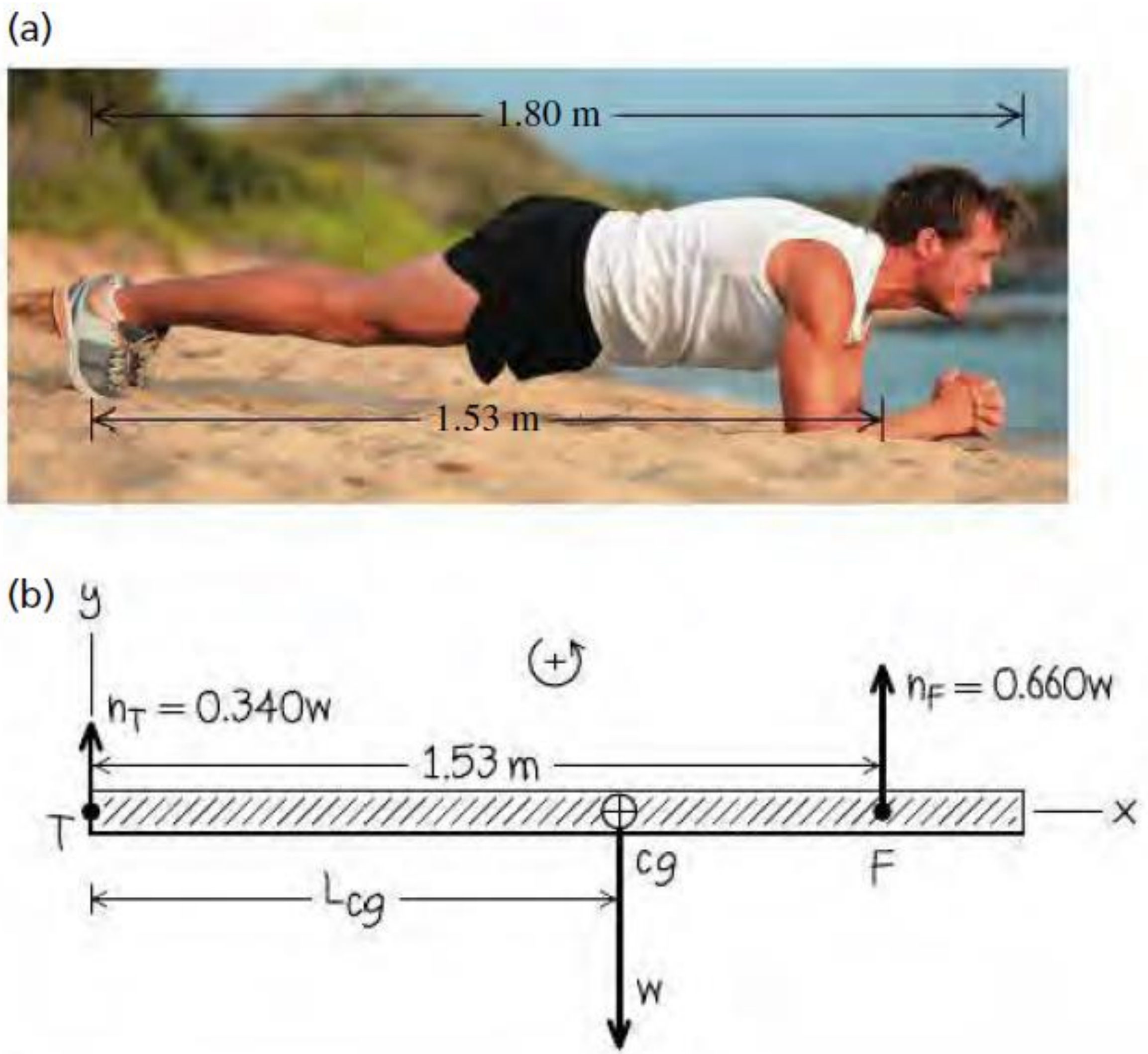
Figure 11.7 At what point will the meter stick with rock attached be in balance?



(ii) In equilibrium, the center of gravity must be at the point of support. Since the rock and meter stick have the same mass and hence the same weight, the center of gravity of the system is midway between their respective centers. The center of gravity of the meter stick alone is 0.50 m from the left end (that is, at the middle of the meter stick), so the center of gravity of the combination of rock and meter stick is 0.25 m from the left end.

ANSWER

例题：

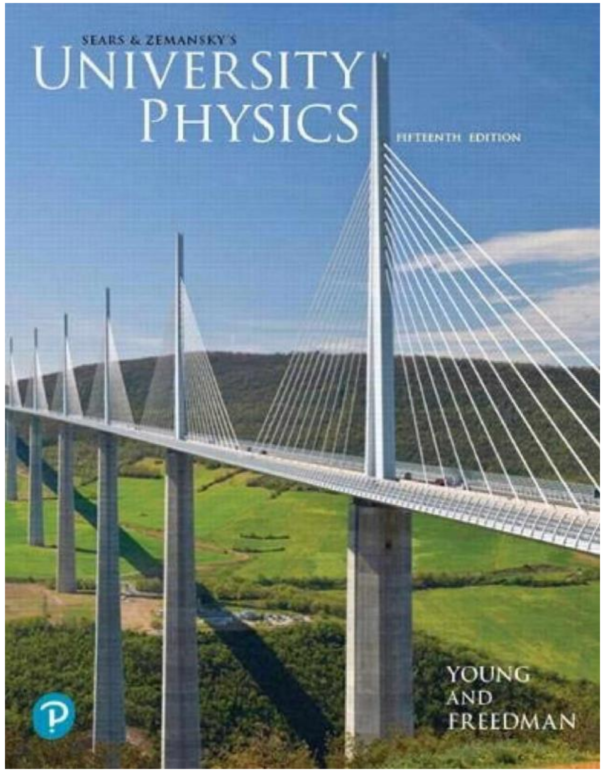


$$\sum \tau_R = 0.340w(0) - wL_{cg} + 0.660w(1.53\ \text{m}) = 0$$

$$L_{cg} = 1.01\ \text{m}$$

普通物理I PHYS1181 第 12 讲 -2

流体力学
Fluid mechanics



A. 流体基本概念

B. 流体静力学

帕斯卡定律

阿基米德定律与浮力

C. 流体动力学

连续性方程

伯努利方程

托里拆利定理

D. *粘滞流体

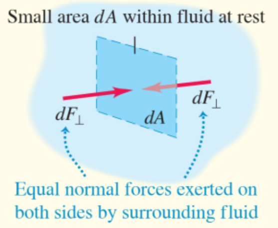
CHAPTER 12 SUMMARY

Density and pressure: Density is mass per unit volume. If a mass m of homogeneous material has volume V, its density ρ is the ratio m/V. Specific gravity is the ratio of the density of a material to the density of water. (See Example 12.1.)

Pressure is normal force per unit area. Pascal's law states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid. Absolute pressure is the total pressure in a fluid; gauge pressure is the difference between absolute pressure and atmospheric pressure. The SI unit of pressure is the pascal (Pa): 1 Pa = 1 N/m^2. (See Example 12.2.)

$$\rho = \frac{m}{V} \quad (12.1)$$

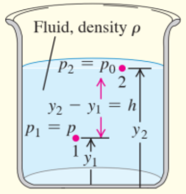
$$p = \frac{dF_\perp}{dA} \quad (12.2)$$



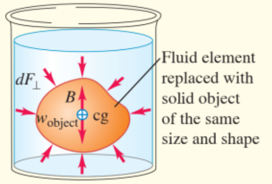
Pressures in a fluid at rest: The pressure difference between points 1 and 2 in a static fluid of uniform density ρ (an incompressible fluid) is proportional to the difference between the elevations y_1 and y_2. If the pressure at the surface of an incompressible liquid at rest is p_0, then the pressure at a depth h is greater by an amount ρgh. (See Examples 12.3 and 12.4.)

$$p_2 - p_1 = -\rho g(y_2 - y_1) \quad (12.5)$$
(pressure in a fluid of uniform density)

$$p = p_0 + \rho gh \quad (12.6)$$
(pressure in a fluid of uniform density)



Buoyancy: Archimedes's principle states that when an object is immersed in a fluid, the fluid exerts an upward buoyant force on the object equal to the weight of the fluid that the object displaces. (See Example 12.5.)



Fluid flow: An ideal fluid is incompressible and has no viscosity (no internal friction). A flow line is the path of a fluid particle; a streamline is a curve tangent at each point to the velocity vector at that point. A flow tube is a tube bounded at its sides by flow lines. In laminar flow, layers of fluid slide smoothly past each other. In turbulent flow, there is great disorder and a constantly changing flow pattern.

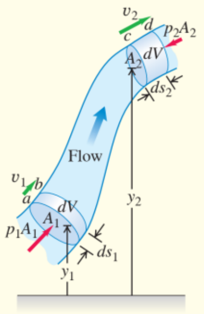
Conservation of mass in an incompressible fluid is expressed by the continuity equation, which relates the flow speeds v_1 and v_2 for two cross sections A_1 and A_2 in a flow tube. The product Av equals the volume flow rate, dV/dt, the rate at which volume crosses a section of the tube. (See Example 12.6.)

Bernoulli's equation states that a quantity involving the pressure p, flow speed v, and elevation y has the same value anywhere in a flow tube, assuming steady flow in an ideal fluid. This equation can be used to relate the properties of the flow at any two points. (See Examples 12.7–12.10.)

$$A_1 v_1 = A_2 v_2 \quad (12.10)$$
(continuity equation, incompressible fluid)

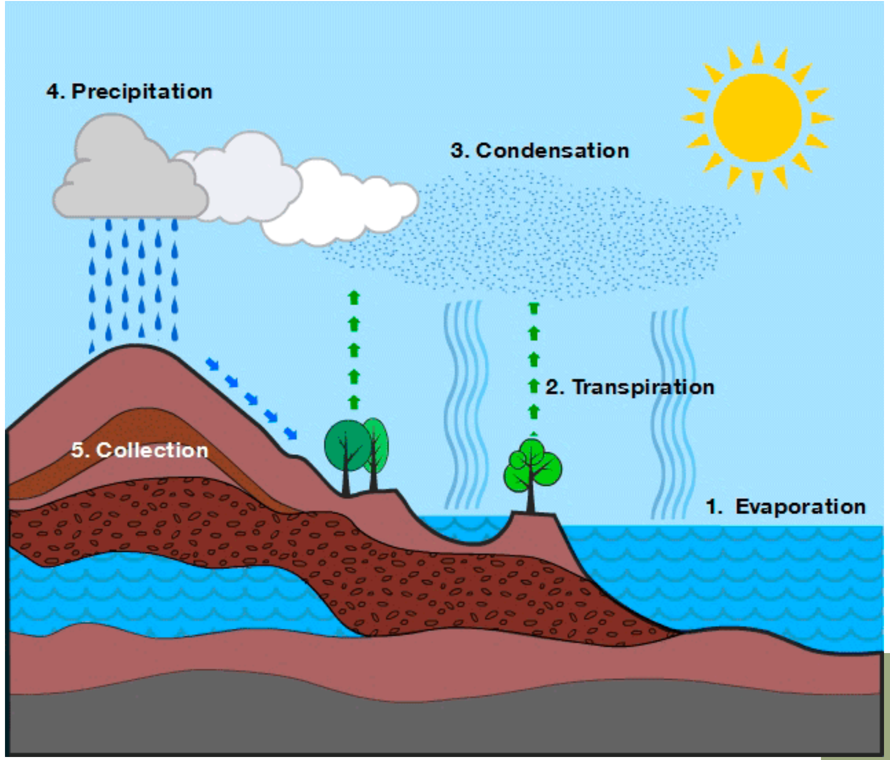
$$\frac{dV}{dt} = Av \quad (12.11)$$
(volume flow rate)

$$p + \rho gy + \frac{1}{2}\rho v^2 = \text{constant} \quad (12.18)$$
(Bernoulli's equation)

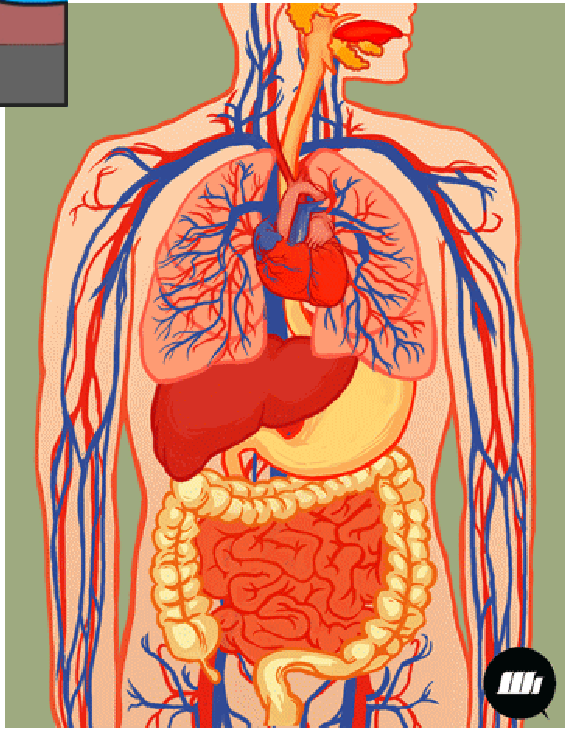


A. 流体（fluid）

A1：什么是流体?



A **fluid** is any substance that can flow and change the shape of the volume that it occupies.
(By contrast, a solid tends to maintain its shape.)



航空航天、水利气象、海洋科学
交通运输、能源发电、生命科学
大禹治水、南水北调、三峡工程、。。。
——流体力学无处不在，已成为除固体力学外研究最多、也是最为重要的力学分支

流体力学包括：流体静力学、流体动力学

人类的发展史，就是一部与水斗争的历史

大前年8月，长江上游洪水，三峡水库出现建库以来最大洪峰，大坝开11孔泄洪

中国新闻组 / 北京20日电 2020-08-20 01:35

前年7月，河南遭遇千年一遇特大暴雨洪水

中国古代繁荣富强的秘密：善于治水

南水北调工程优化了水资源配置。工程累计调水量近300亿立方米，直接受益人口超过1.2亿人，成为多个重要城市生活用水的主力水源。南水北调为京津冀协同发展、雄安新区建设等国家重大战略实施提供了可靠的水资源支撑。 “南水”占到北京中心城区自来水供水量七成以上，天津市14个行政区全部用上“南水”，河南多个城市主城区100%使用“南水”，河北石家庄、邯郸、沧州等市的90多个县区受益，提升了江苏苏中苏北地区防洪排涝抗旱能力。

这两天秦岭火了！

南方的水气怎么才能翻过秦岭?

超级水利工程“引汉济渭”——南北凿穿秦岭山脉！

98公里超长隧洞，最深处2012米

A2：气体、液体、密度

流体主要包括：气体（gases）和液体（liquids）

密度定义：

$$\rho = \frac{\Delta m}{\Delta V}.$$

对于均匀物质：

Density of a homogeneous material → $\rho = \dfrac{m}{V}$ ← m: Mass of material; V: Volume occupied by material

常见物质的密度

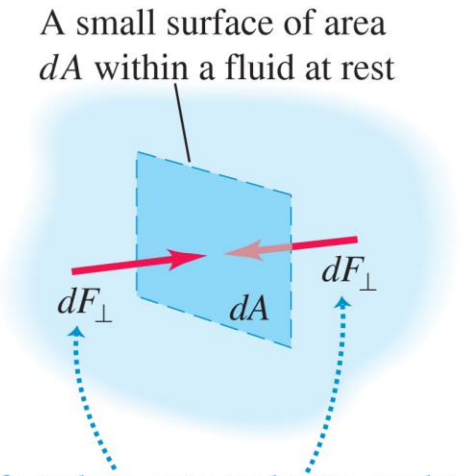
Material or Object	Density (kg/m^3)
Interstellar space	10^{-20}
Best laboratory vacuum	10^{-17}
Air: 20°C and 1 atm pressure	1.21
20°C and 50 atm	60.5
Styrofoam	1×10^2
Ice	0.917×10^3
Water: 20°C and 1 atm	0.998×10^3
20°C and 50 atm	1.000×10^3
Seawater: 20°C and 1 atm	1.024×10^3
Whole blood	1.060×10^3
Iron	7.9×10^3
Mercury (the metal, not the planet)	13.6×10^3
Earth: average	5.5×10^3
core	9.5×10^3
crust	2.8×10^3
Sun: average	1.4×10^3
core	1.6×10^5
White dwarf star (core)	10^{10}
Uranium nucleus	3×10^{17}
Neutron star (core)	10^{18}

B. 流体静力学

B1：流体静力学：压强

流体静止时，不存在任何切向力（否则就会流动），
所以我们可以得到：

流体静力学第一结论：流体内压力总是垂直于流体内任意一个面



The surface does not accelerate, so the surrounding fluid exerts equal normal forces on both sides of it. (The fluid cannot exert any force parallel to the surface, since that would cause the surface to accelerate.)

流体中某一点处的压强等于：

Pressure at a point in a fluid → $$p = \frac{dF_{\perp}}{dA}$$
$dF_{\perp}$ ← Normal force exerted by fluid on a small surface at that point
dA ← Area of surface

若在该点所在的某一平面上处处压力均匀（比如重力作用下的流体中等高的平面），则有：

$$p = \frac{F_{\perp}}{A}$$

压强是个标量，其单位：

$$1 \text{ pascal} = 1 \text{ Pa} = 1 \text{ N/m}^2$$

大气压：

$$(p_a)_{av} = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$
$$= 1.013 \text{ bar} = 1013 \text{ millibar}$$
$$= 760 \text{ torr} = 14.7 \text{ lb/in.}^2$$

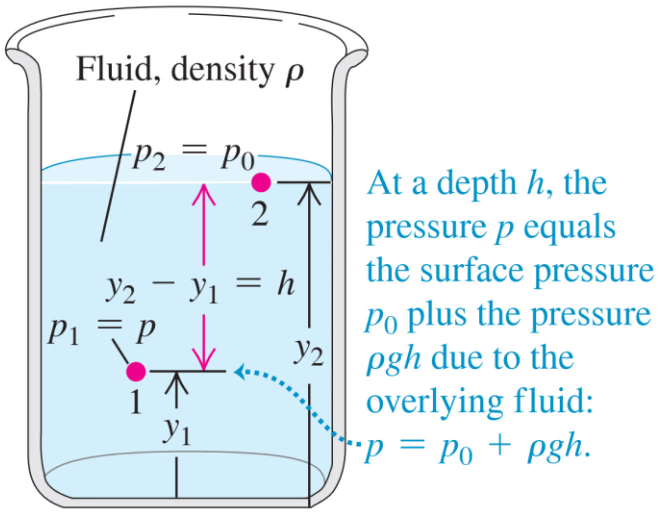
流体静力学第二结论：流体内某点处压强在任意方向上都相同

流体静力学第三结论：重力作用下的流体平衡方程

Pressure at depth *h* in a fluid of uniform density

$$p = p_0 + \rho g h$$

- p_0: Pressure at surface of fluid
- ρ: Uniform density of fluid
- g: Acceleration due to gravity ($g > 0$)
- h: Depth below surface

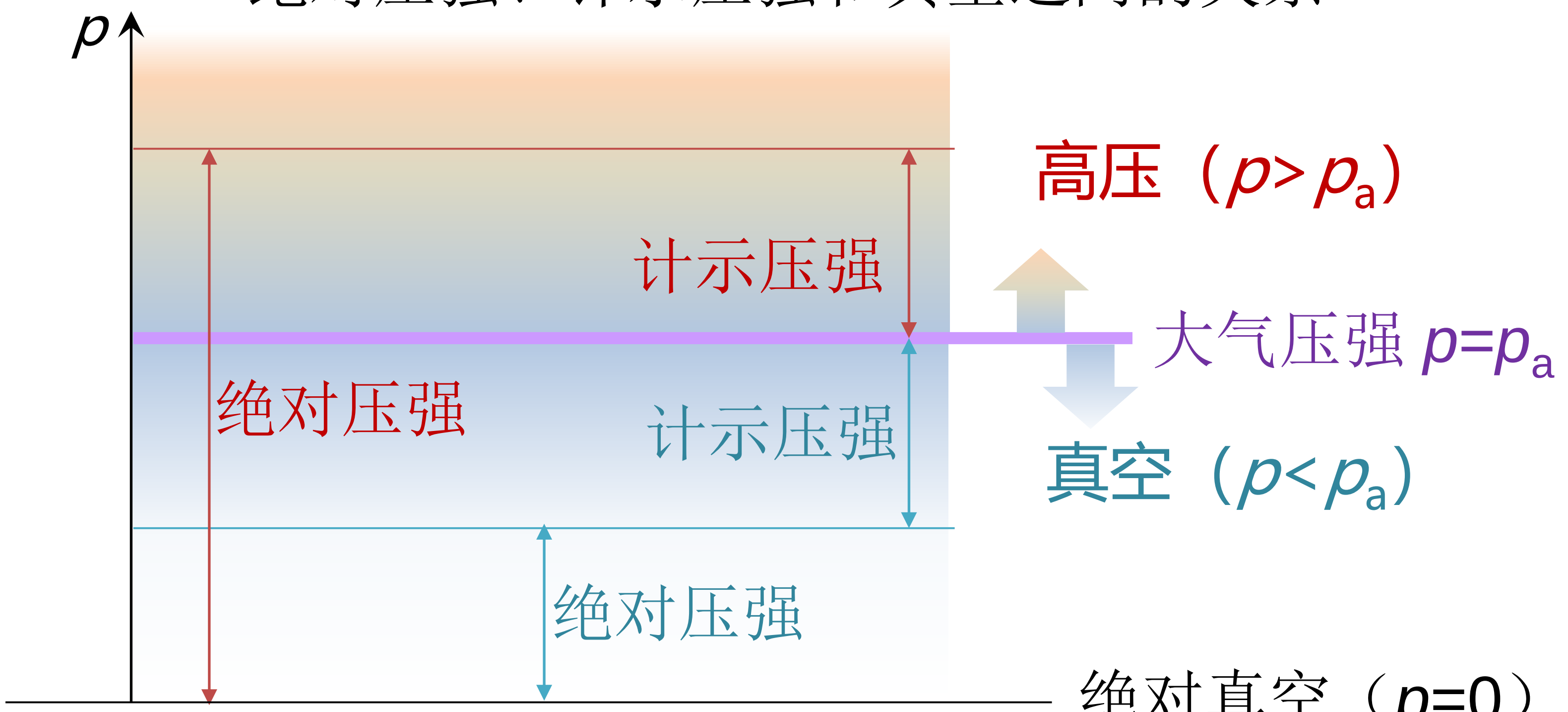


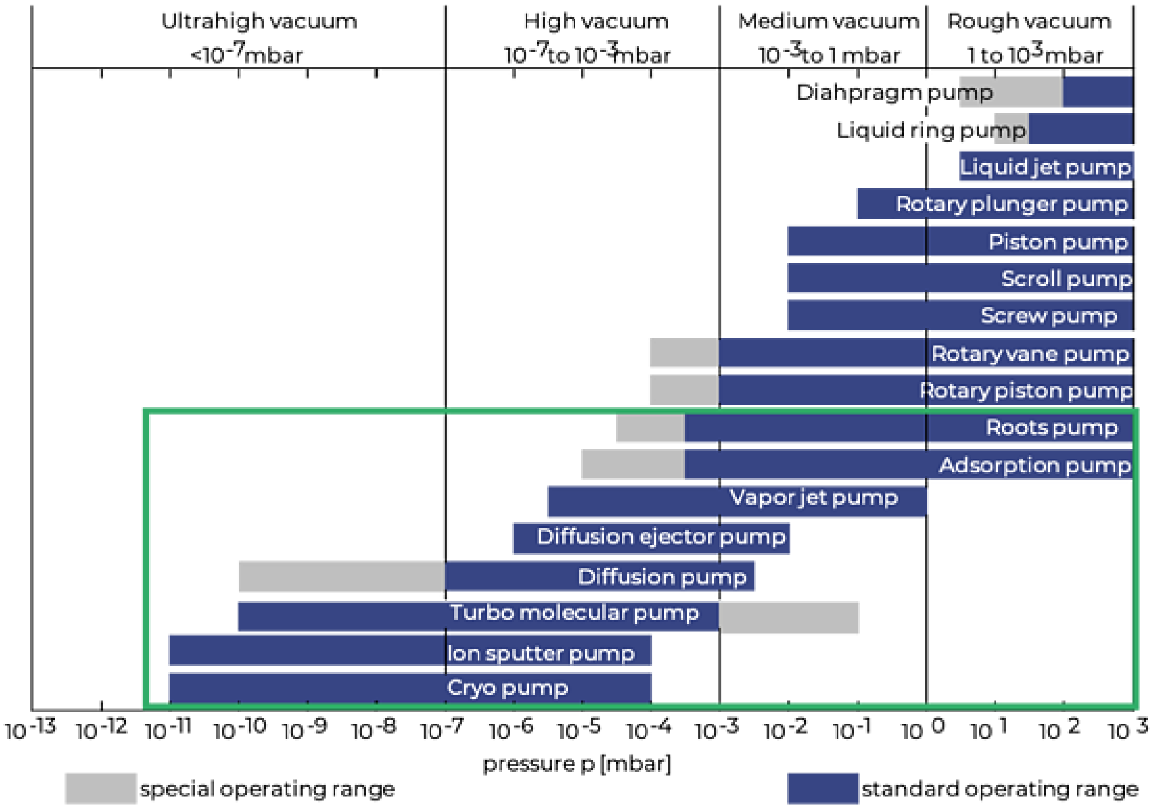
Pressure difference between levels 1 and 2:
$$p_2 - p_1 = -\rho g (y_2 - y_1)$$
The pressure is greater at the lower level.

B2：压强的测量

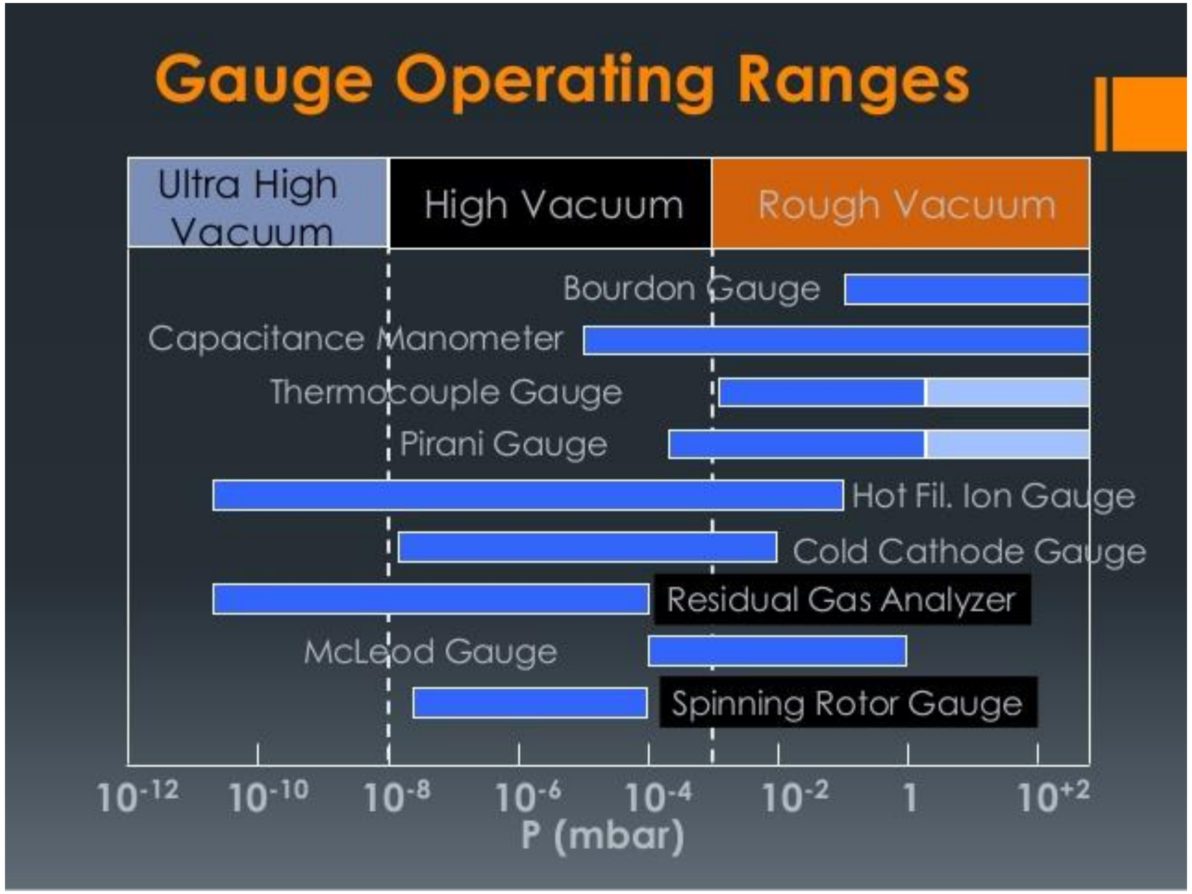
- 绝对压强（absolute pressure）
- 计示压强（gauge pressure）（表压）

绝对压强、计示压强和真空之间的关系





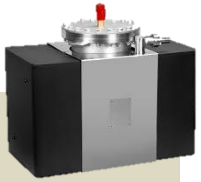
真空的获取：真空泵



真空度（压强）的测量：真空规

超高真空腔室

工作真空10^{-10} mbar



离子泵

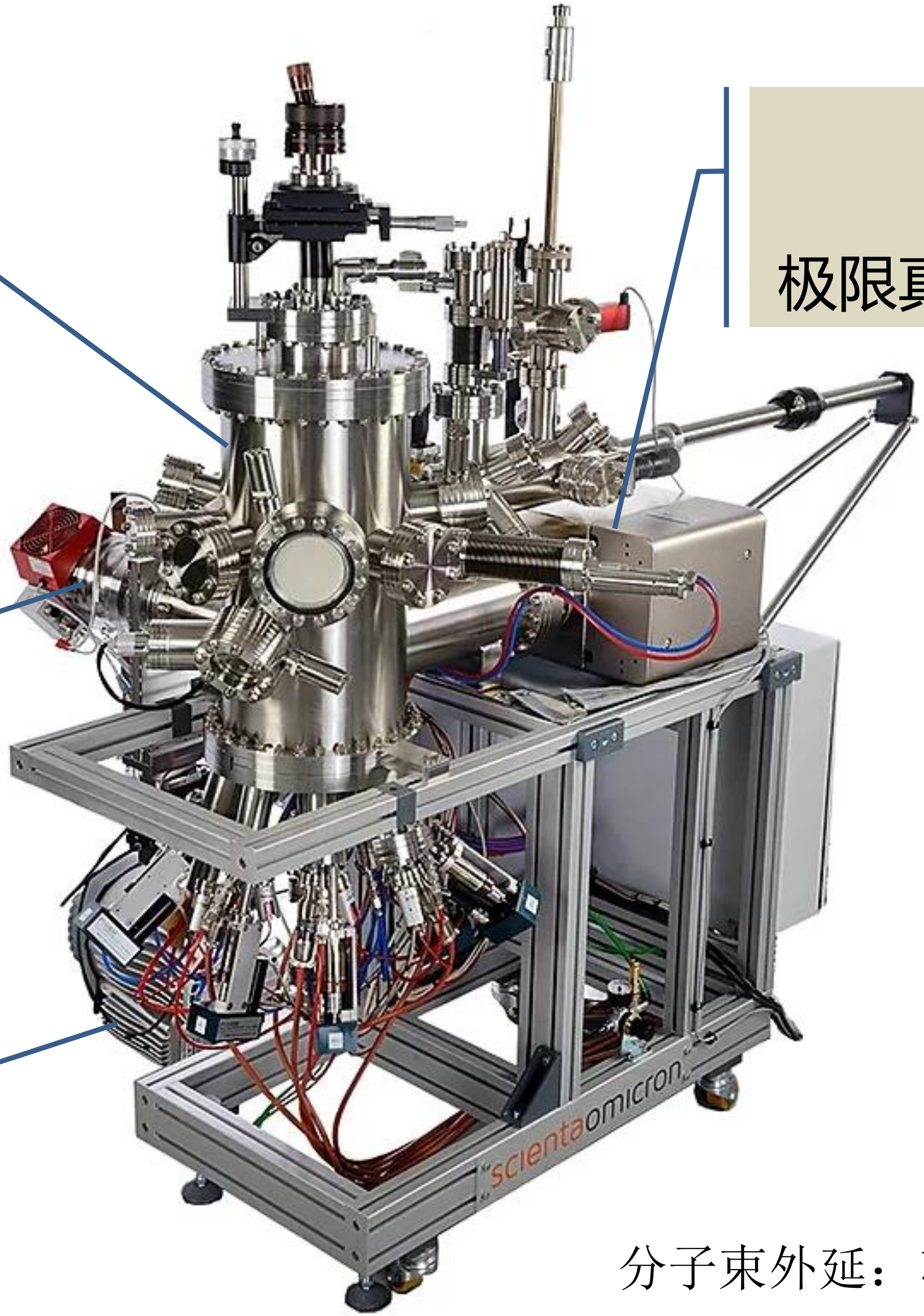
极限真空10^{-11} mbar

涡轮分子泵

极限真空10^{-9} mbar

机械泵

极限真空10^{-3} mbar



分子束外延：芯片的核心技术

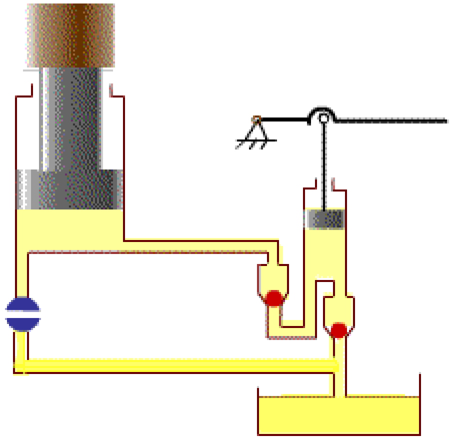
B3：帕斯卡原理（Pascal’s principle）

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

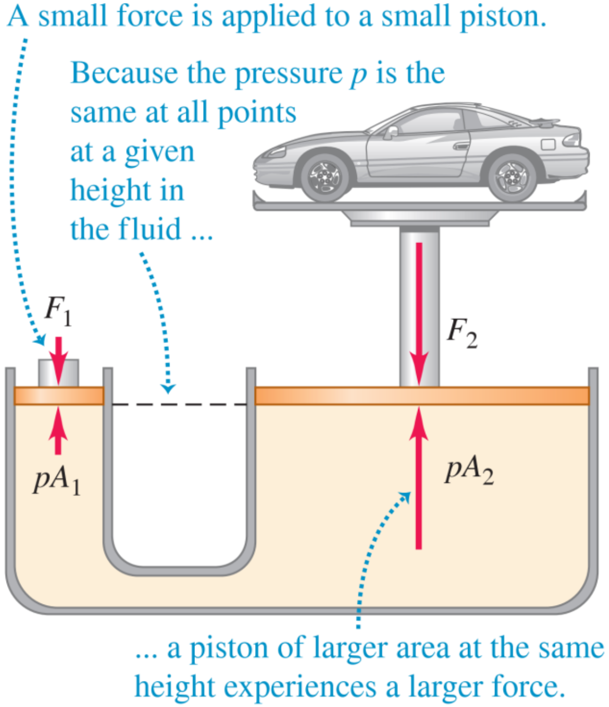
帕斯卡原理：封闭容器中的不可压缩流体的某一部分施加的压强变化，将毫无损失地传递至流体的各个部分和容器壁。

根据帕斯卡原理，在液体系统中的一个活塞上施加一定的压强，必将在另一个活塞上产生相同的压强增量。如果第二个活塞的面积是第一个活塞的面积的10倍，那么作用于第二个活塞上的力将增大为第一个活塞的10倍，而两个活塞上的压强仍然相等。

Heimlich maneuver
（海姆利克操作法）



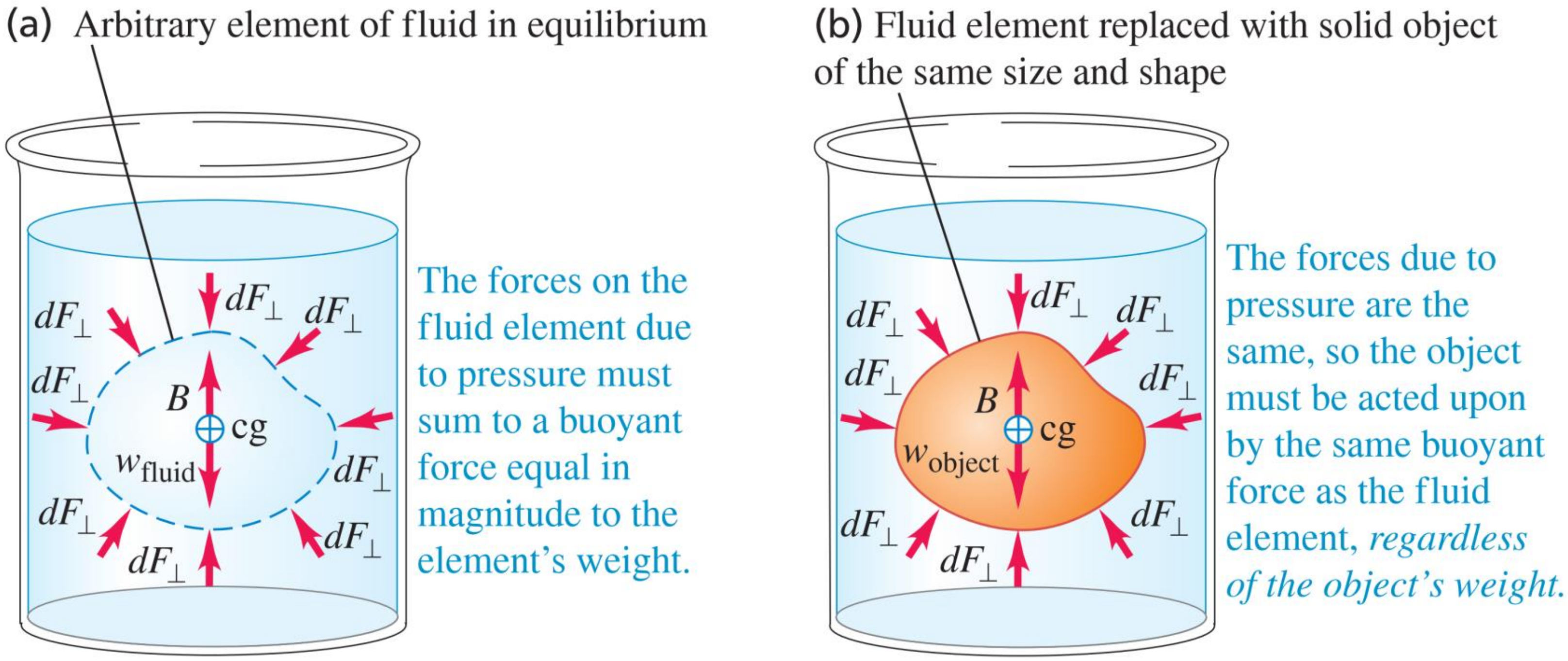
hydraulic lift（液压千斤顶）



A6：阿基米德原理（Archimedes’ principle）与浮力

When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body.

阿基米德原理：浸在流体中的物体受到竖直向上的浮力，其大小等于物体所排开流体的重力。



C. 流体动力学

真实流体的运动（流动）极为复杂，并且尚未完全得到理解。因此，我们在研究流体运动的时候，先讨论最简单的**理想流体**的流动，主要有四个假设：

1、定常流动（任一点流速不随时间发生变化）

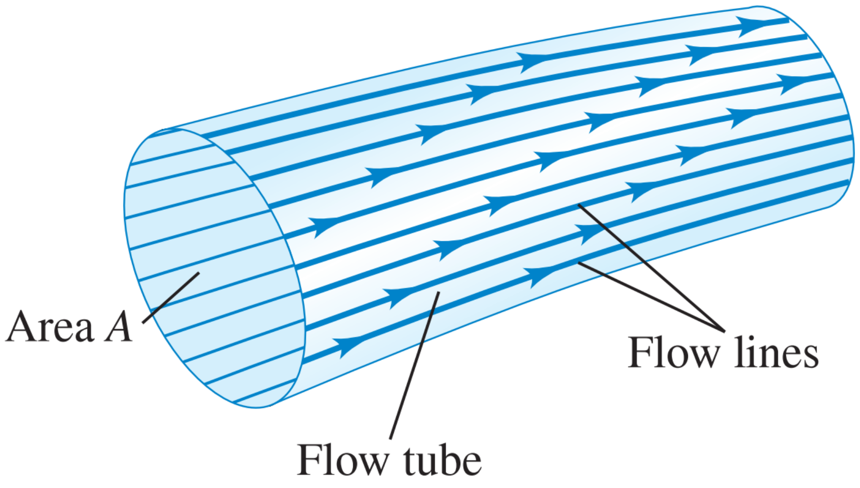
2、不可压缩

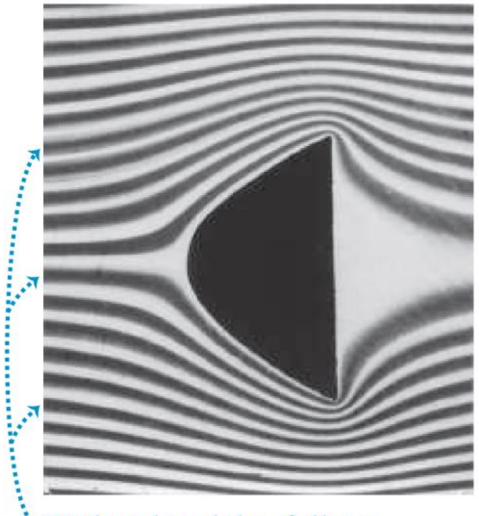
3、无粘性

4、无旋转

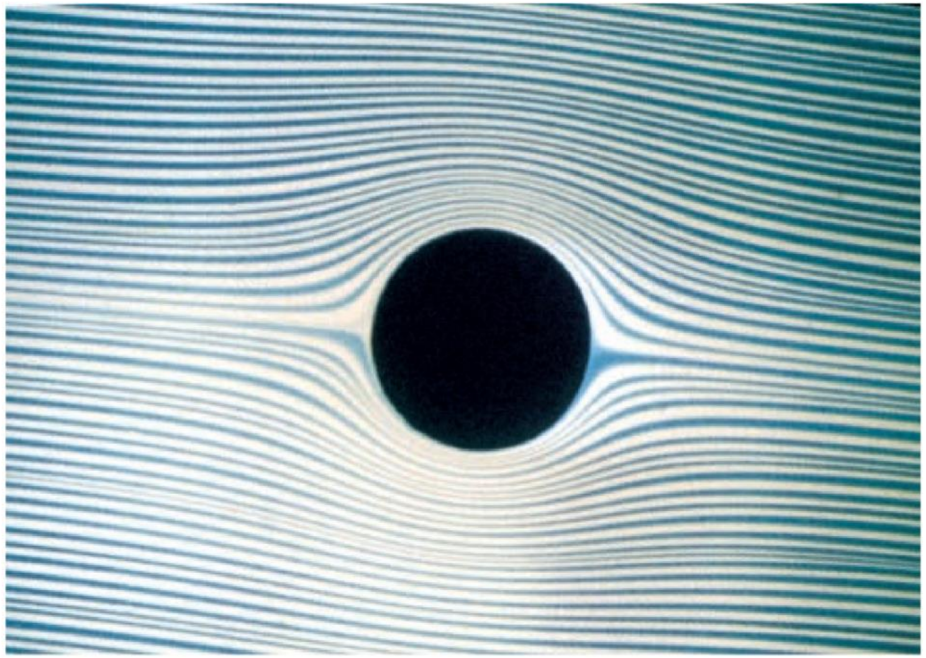
C2：流管、流线（flow tube, flow lines)

Figure **12.18** A flow tube bounded by flow lines. In steady flow, fluid cannot cross the walls of a flow tube.





Dark-colored dye follows streamlines of laminar flow (flow is from left to right).



Courtesy D. H. Peregrine, University of Bristol

汽车风洞实验

C2：连续性方程（continuity equation）

前提：不可压缩 → 密度不变
+
物质守恒 → 连续性方程

$$\rho A_1 v_1\, dt = \rho A_2 v_2\, dt$$

连续性方程：

Continuity equation for an incompressible fluid:

$$A_1 v_1 = A_2 v_2$$

Cross-sectional area of flow tube at two points (see Fig. 12.21)

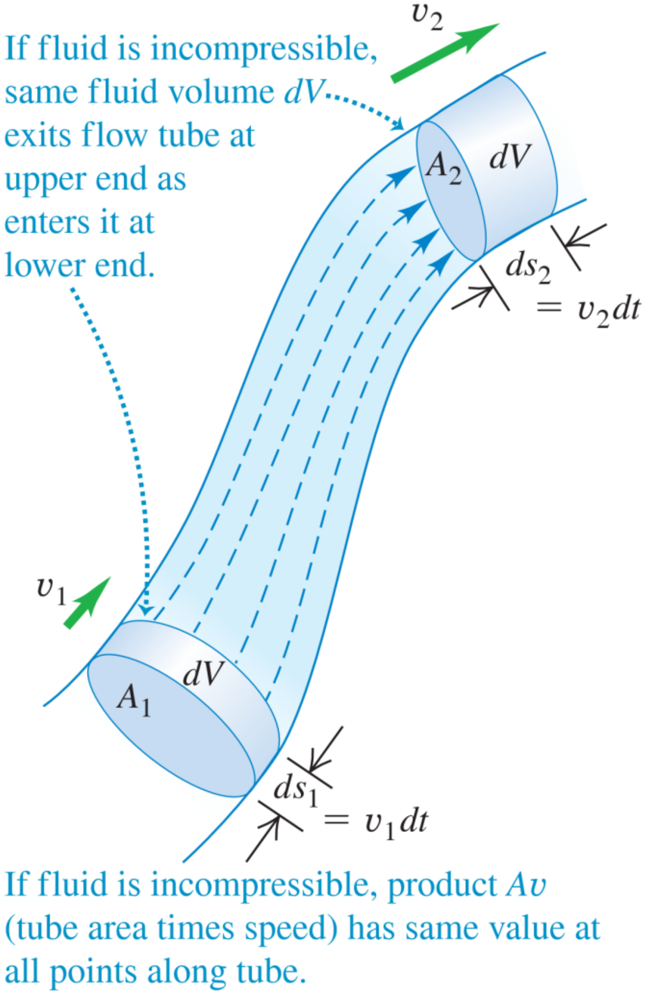
Speed of flow at the two points

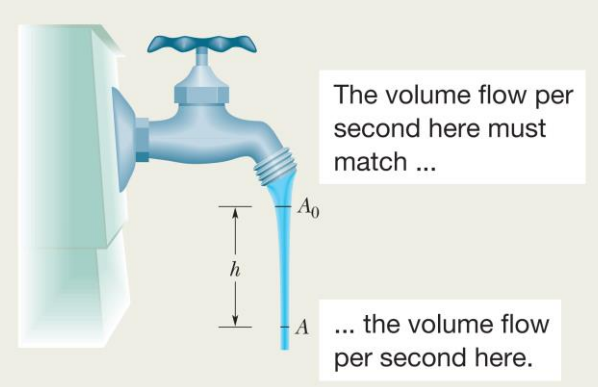
体积流速：

$$R_V = Av = \text{a constant} \quad \text{(volume flow rate, equation of continuity)},$$

质量流速：

$$R_m = \rho R_V = \rho A v = \text{a constant} \quad \text{(mass flow rate).}$$

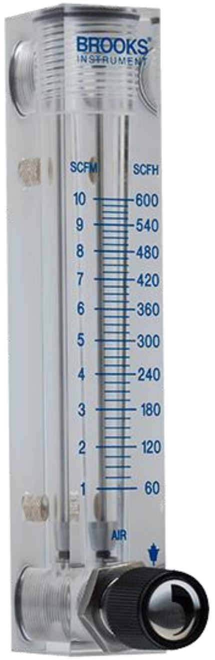


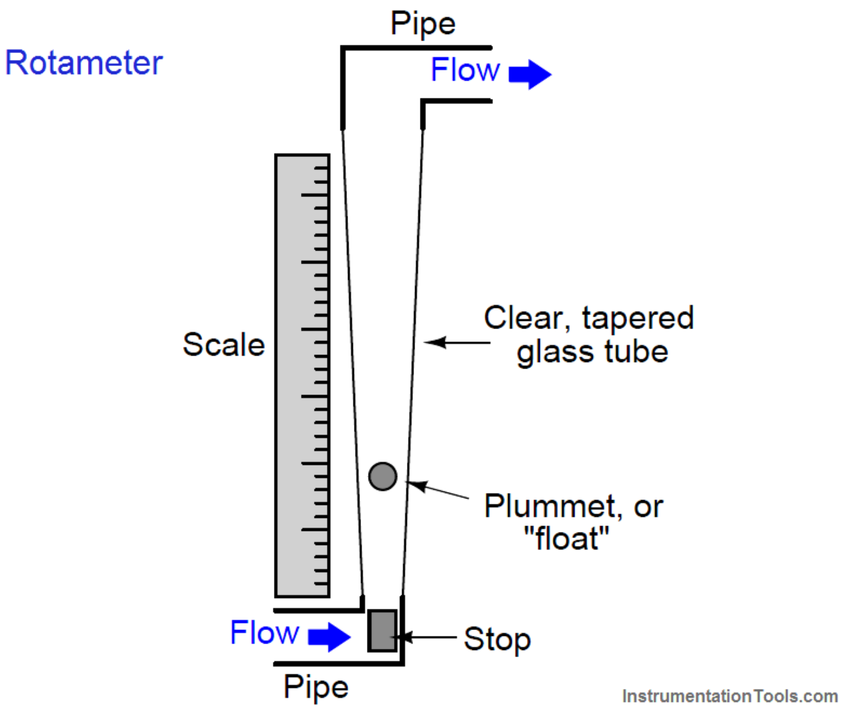


实际应用中，由于流体压强是可变的，流速往往不是常数，需要测量和控制流量（单位时间内的流量）：

浮子流量计（可变面积流量计、**Rotameter**）可以用来测量流速，是最简单快捷的流量测量计。

在工业加工和科研中，稳定流速的气源非常重要，例如芯片刻蚀、化学化工、晶体生长等。因此，需要高精度的**质量流量计**（**Mass flow controller, MFC**）





C3：伯努利方程（Bernoulli equation）

根据能量守恒，压强对流体做的功： $dW = dK + dU$

其中：

$$dW = p_1 A_1\, ds_1 - p_2 A_2\, ds_2 = (p_1 - p_2)dV$$

$$dK = \frac{1}{2}\rho\, dV(v_2^2 - v_1^2)$$

$$dU = \rho\, dV\, g(y_2 - y_1)$$

$$dV = A_1\, ds_1 = A_2\, ds_2$$

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

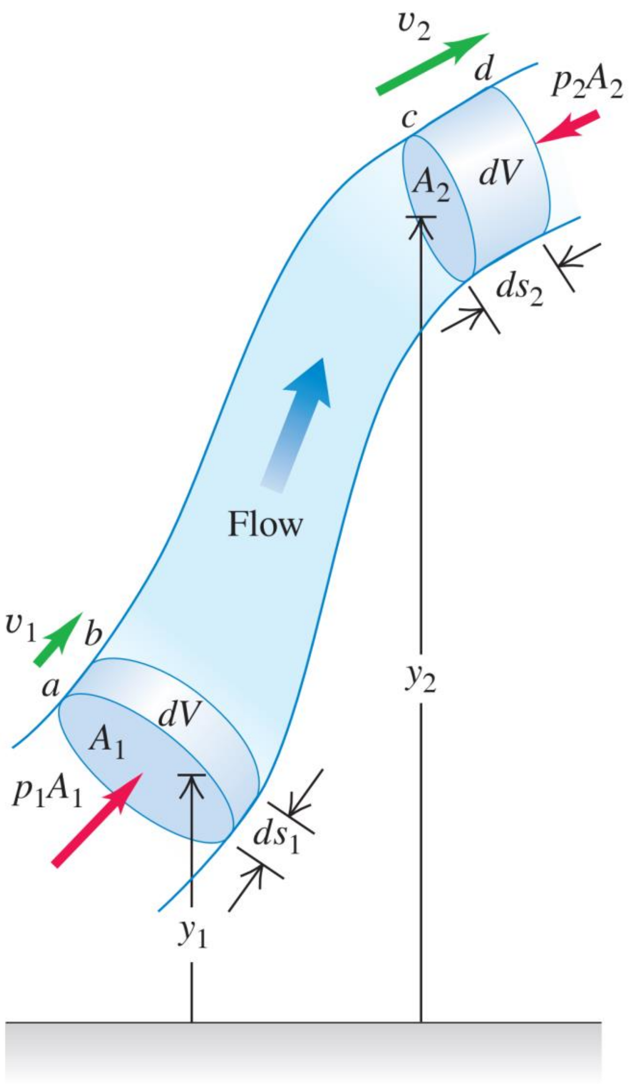
可得：**伯努利方程：**

Bernoulli's equation for an ideal, incompressible fluid:

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

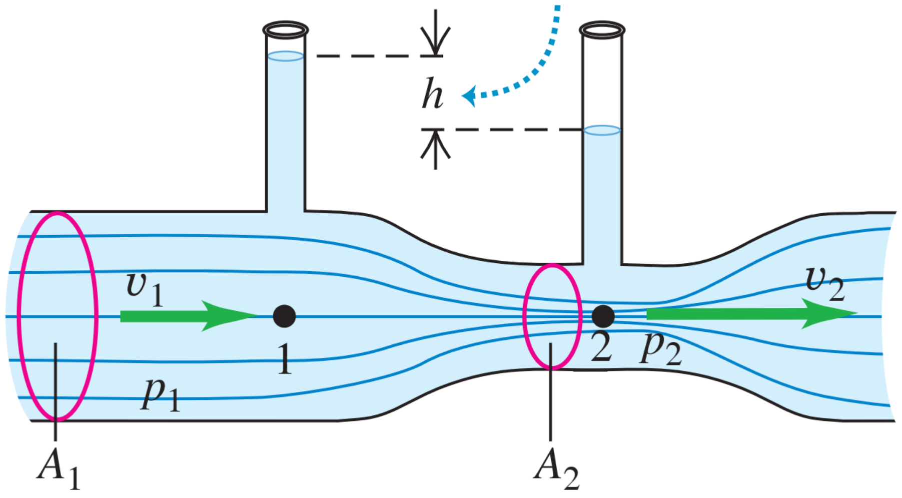
- Pressure: p
- Fluid density: ρ
- Acceleration due to gravity: g
- Elevation: y
- Flow speed: v
- constant: Value is **same** at all points in flow tube.

Figure 12.23 Deriving Bernoulli's equation. The net work done on a fluid element by the pressure of the surrounding fluid equals the change in the kinetic energy plus the change in the gravitational potential energy.



C4：伯努利方程的应用：

(a) Venturi meter（文丘里流量计）：



$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 = (A_1/A_2)v_1$$

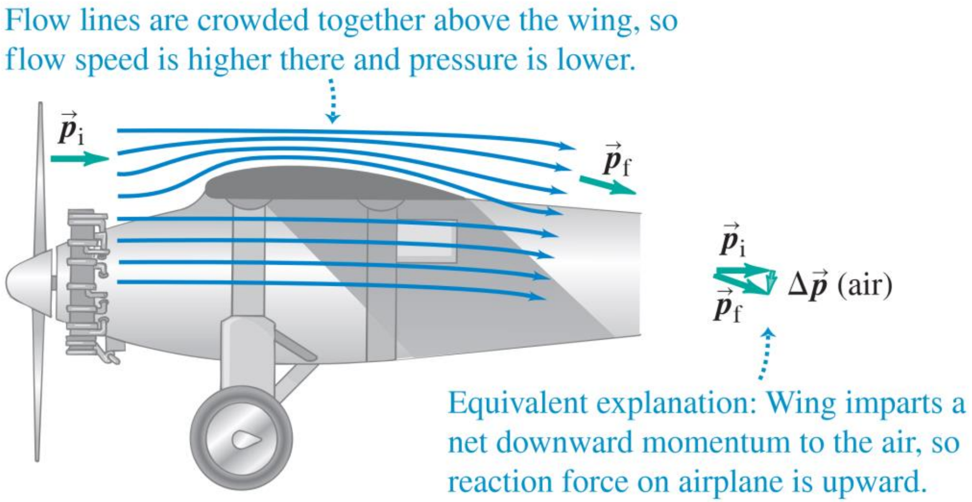
$$p_1 - p_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$$

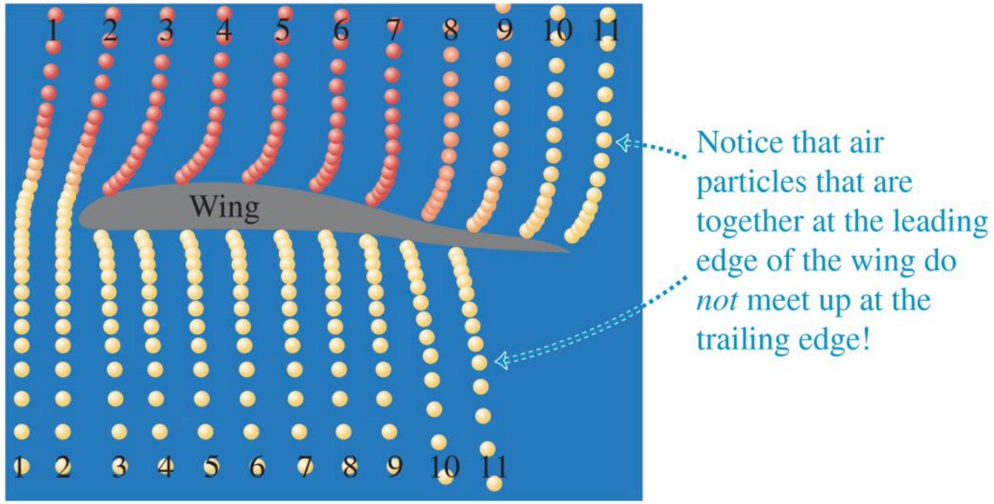
有点反直觉：流速大的地方，压强小

(b) 飞机机翼的升力：

(a) Flow lines around an airplane wing



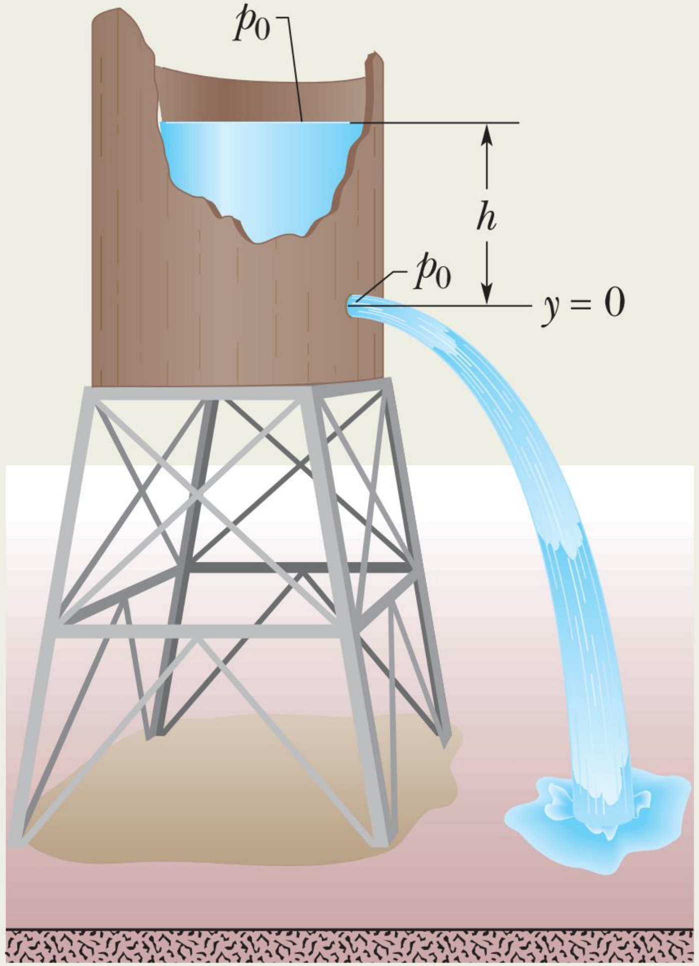
(b) Computer simulation of air parcels flowing around a wing, showing that air moves much faster over the top than over the bottom



*实际上伯努利原理只是飞机产生升力的部分原因。机翼的具体形状、攻角等都对飞机的升力有作用。

(c) 托里拆利定理：

小孔面积a，出水速度v，水箱截面积A，水位下降速度v_0



$$R_V = av = Av_0$$

$$v_0 = \frac{a}{A} v.$$

$$a \ll A,\ v_0 \ll v.$$

$$p_0 + \frac{1}{2}\rho v_0^2 + \rho g h = p_0 + \frac{1}{2}\rho v^2 + \rho g(0).$$

因此，小孔出水速度： $v = \sqrt{2gh}.$

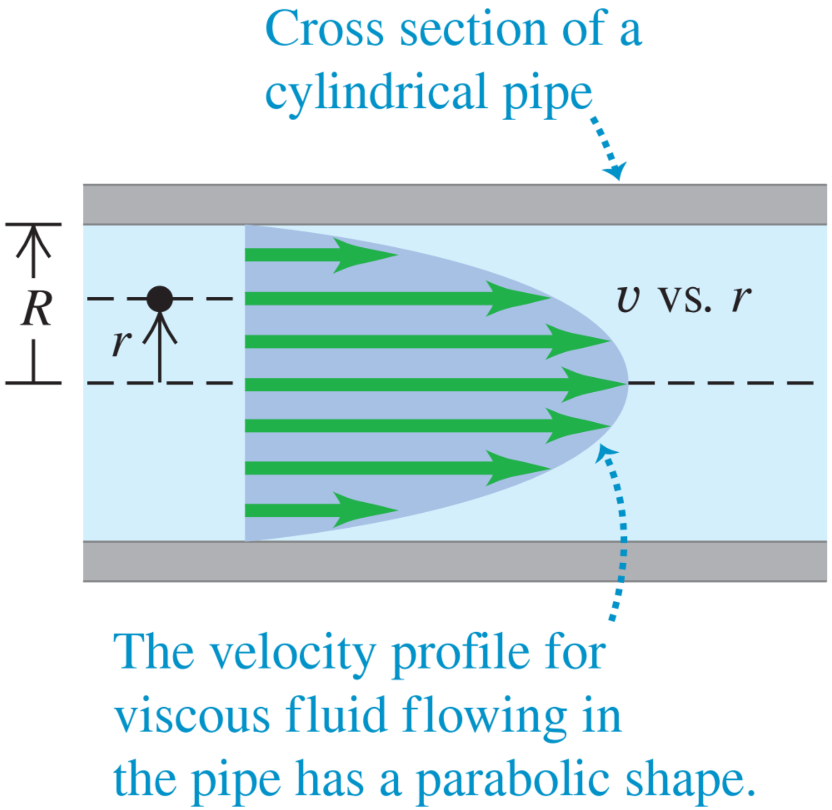
*D. 粘滞流体

实际的流体都具有粘滞性，即使是水、空气

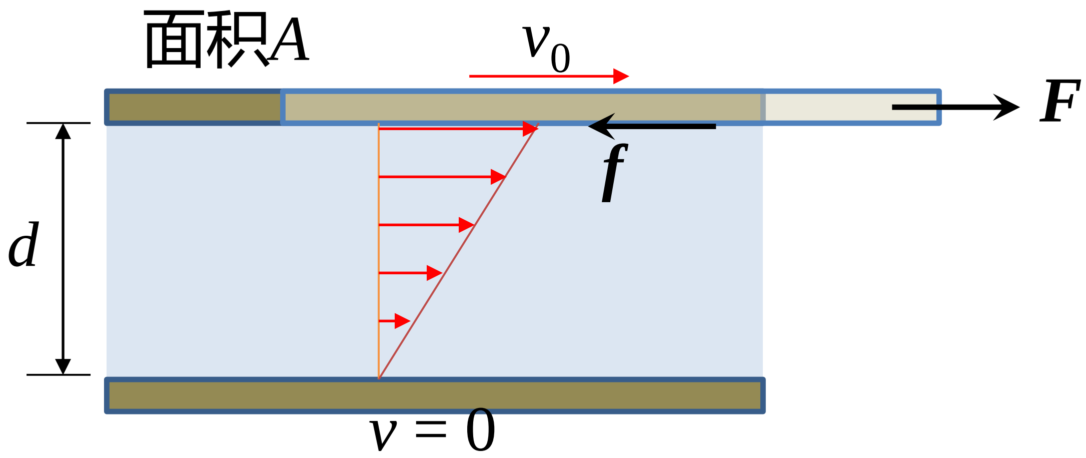
典型的粘滞流体的例子：

鸡蛋清、麦芽糖、蜂蜜

岩浆、柏油、胶水



D1：粘滞系数



考虑实验：两块间距为 d 中间夹有流体的固体板面，保持下板固定，以拉力 F 水平拉上板使其以低速 v_0 直线运动。

上板除了受到拉力 F 外，还受到液体的粘滞阻力 f，大小相等，方向相反。

对于**牛顿流体**，粘滞力与速度成正比，有牛顿粘滞定律：

粘滞力 f 的大小与板的面积A、v_0/d：

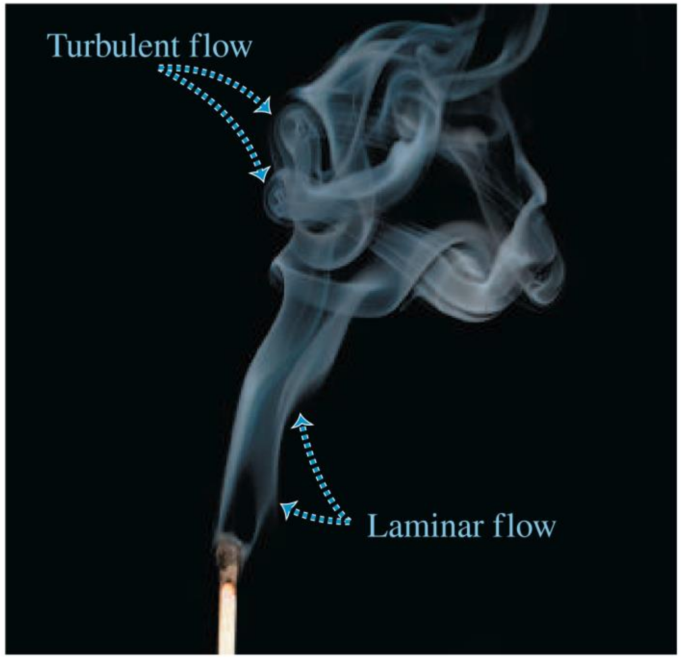
$$\frac{f}{A} = \eta \frac{v_0}{d}$$

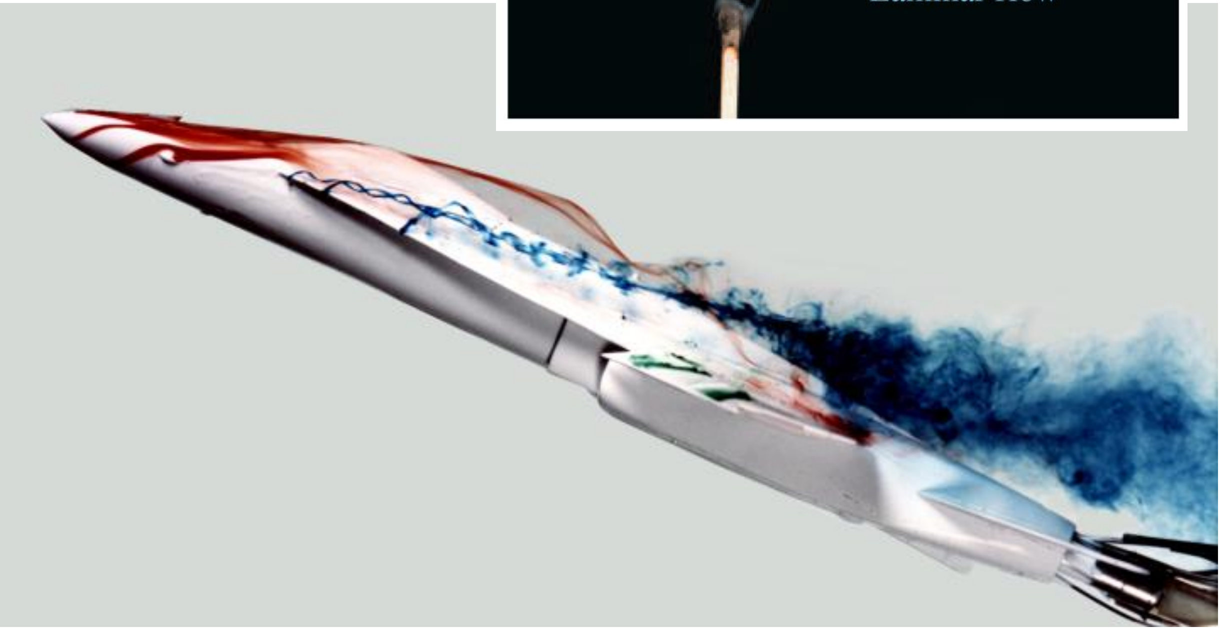
不符合牛顿粘滞定律的流体为非牛顿流体。

D2：层流与湍流（laminar flow, turbulent flow）

(a) Low speed:
laminar flow

(b) High speed:
turbulent flow

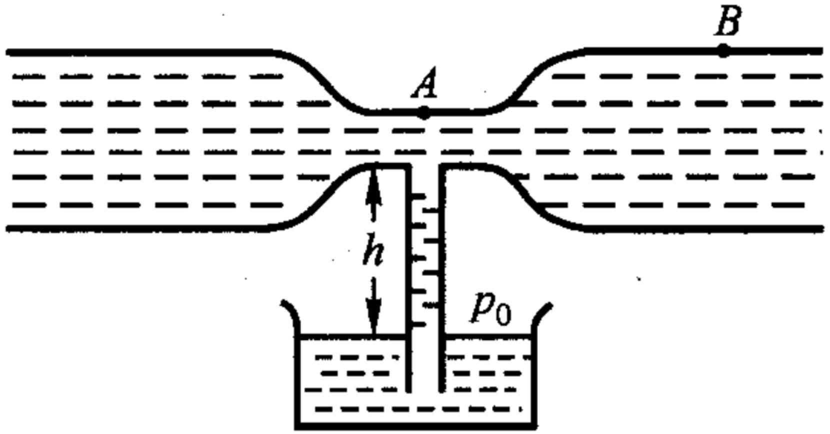




HW#1 液体在一水平管道中流动，A 处和 B 处的横截面积分别为 S_A 和 S_B. B 管口与大气相通，压强为 p_0. 若在 A 处用一细管与容器相通，如图所示，试证明，当 h 满足关系：

$$h = \frac{Q^2}{2g}\left(\frac{1}{S_A^2} - \frac{1}{S_B^2}\right)$$

时，A 处的压强刚好能将从比水平管低 h 处的同种液体吸上来. 其中 Q 为体积流量.



HW#2 一倒立的圆锥形容器，高为 h，底面半径为 R，容器内装满水，下方锥顶角处有一面积为 S 的小孔，水从小孔中流出. 试求水面下降到 $\frac{h}{2}$ 高度时所需的时间.